

# UPDATED RESULTS ON THE CKM MATRIX AND THE UNITARITY TRIANGLE

Including results presented up to  
ICHEP 06, Moscow, Russia  
and BEAUTY 06, Oxford, England

*P r e l i m i n a r y*

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The CKMfitter Group

## Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach *Rfit*. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

*CP* VIOLATION AND THE CKM MATRIX:  
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES

By CKMfitter Group

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## References

- [1] Results presented at the San Diego CKM workshop, <http://ckm2005.ucsd.edu/hep-ph/0512039>
- [2] Particle Data Group (W.-M. Yao *et al.*), *Journal of Physics G* **33**, 1 (2006)
- [3] The Heavy Flavor Averaging Group (HFAG), Summer 2006 averages, <http://www.slac.stanford.edu/xorg/hfag/>
- [4] For the inclusive average we are taking the BLNP number. The DGE result is very close to. The theoretical error on the inclusive average is obtained by adding linearly the contributions from weak annihilation, subleading shape functions and the HQE uncertainty on  $m_b$ .  
We use only branching fractions measured for  $B \rightarrow \pi \ell \nu$ . we average the results obtained from the two unquenched Lattice calculations and the LCSR calculation for the form factor quoted by HFAG [3] in such a way that the smallest theoretical error is kept.  
Also for the average between the inclusive and exclusive result we keep the smallest theoretical error.
- [5] O. Buchmüller and H. Flächer, Fit to Moment Measurements from  $B \rightarrow X_c \ell \nu$  and  $B \rightarrow X_s \gamma$  Decays using Heavy Quark Expansions in the Kinetic Scheme, *Phys. Rev. D* **73**, 073008 (2006) (hep-ph/0507253 (2005))
- [6] Average of  $\eta_{+-}$  between KTeV and KLOE:  
KTeV collaboration (T. Alexopoulos *et al.*), *Phys. Rev. D*, volume 70 092006,  
KLOE collaboration (F. Ambrosino *et al.*), hep-ex/0603041 (2006)  
 $\varepsilon_K$  calculated from  $\eta_{+-}$  and  $Re(\epsilon'/\epsilon)$  [2]
- [7] For this only the recent CDF measurement (hep-ex/0609040) has been used as it currently dominates the world average which still needs to be determined by HFAG.
- [8] B. Aubert *et al.*, BABAR-CONF-06/037, hep-ex/0608002 (2006). Belle also reported a Dalitz and an isospin (pentagon) analysis (hep-ex/0609003), but these results are not used in our Summer 06 results. The averaging is in progress.
- [9] The Heavy Flavor Averaging Group (HFAG), Winter 2006 averages, <http://www.slac.stanford.edu/xorg/hfag/>
- [10] The CDF Collaboration, the D0 Collaboration, and the Tevatron Electroweak Working Group, Combination of CDF and D0 Results on the Top-Quark Mass, hep-ex/0603039 (2006)
- [11] B. Aubert *et al.*, BABAR-CONF-06/028, hep-ex/0608019 (2006)  
. Belle Collaboration, T. Browder's talk at ICHEP 06:  
[http://ichep06.jinr.ru/reports/287\\_8s5\\_15p18\(1\).ppt](http://ichep06.jinr.ru/reports/287_8s5_15p18(1).ppt)
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- [13] U. Nierste, private communication (2003)
- [14] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996)

Parameter	Value $\pm$ Error(s)	Reference	Errors	
			GS	TH
$ V_{ud} $ (nuclei)	$0.97377 \pm 0.00027$	[1]	*	-
$ V_{us} $ ( $K_{\ell 3}$ )	$0.2257 \pm 0.0021$	[2]	*	-
$ V_{ub} $	$(4.10 \pm 0.09 \pm 0.39) \times 10^{-3}$	[3, 4]	*	*
$ V_{cb} $ (incl.)	$(41.70 \pm 0.70) \times 10^{-3}$	[2]	*	-
$ V_{cb} $ (excl.)	$(39.7 \pm 2.0) \times 10^{-3}$	[3]	*	-
$ \varepsilon_K $	$(2.221 \pm 0.008) \times 10^{-3}$	[6]	*	-
$\Delta m_d$	$(0.507 \pm 0.004) \text{ ps}^{-1}$	[3]	*	-
$\Delta m_s$	CDF measurement	[7]	*	-
$\sin(2\beta)_{[c\bar{c}]}$	$0.675 \pm 0.026$	[3]	*	-
$S_{\pi\pi}^{+-}$	$-0.59 \pm 0.09$	[3]	*	-
$C_{\pi\pi}^{+-}$	$-0.39 \pm 0.07$	[3]	*	-
$C_{\pi\pi}^{00}$	$-0.36^{+0.33}_{-0.31}$	[3]	*	-
$B_{\pi\pi}$ all charges	Inputs to isospin analysis	[3]	*	-
$S_{\rho\rho,L}^{+-}$	$-0.13 \pm 0.19$	[3]	*	-
$C_{\rho\rho,L}^{+-}$	$-0.06 \pm 0.14$	[3]	*	-
$B_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[3]	*	-
$B^0 \rightarrow (\rho\pi)^0 \rightarrow 3\pi$	Time-dependent Dalitz analysis	[8]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to GLW analysis	[9]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to ADS analysis	[9]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	GGSZ Dalitz analyses	[9]	*	-
$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau)$	Experimental likelihoods	[11]	*	-
$\bar{m}_c(m_c)$	$(1.24 \pm 0.037 \pm 0.095) \text{ GeV}$	[5]	*	*
$\bar{m}_t(m_t)$	$(162.3 \pm 2.2) \text{ GeV}$	[10]	*	-
$m_{K^+}$	$(493.677 \pm 0.016) \text{ MeV}$	[2]	-	-
$\Delta m_K$	$(3.4833 \pm 0.0066) \times 10^{-12} \text{ MeV}$	[2]	-	-
$m_{B_d}$	$(5.2794 \pm 0.0005) \text{ GeV}$	[2]	-	-
$m_{B_s}$	$(5.3696 \pm 0.0024) \text{ GeV}$	[2]	-	-
$m_W$	$(80.423 \pm 0.039) \text{ GeV}$	[2]	-	-
$G_F$	$1.16639 \times 10^{-5} \text{ GeV}^{-2}$	[2]	-	-
$f_K$	$(159.8 \pm 1.5) \text{ MeV}$	[2]	-	-
$B_K$	$0.79 \pm 0.04 \pm 0.09$	[1]	*	*
$\alpha_s(m_Z^2)$	$0.1176 \pm 0.0020$	[2]	-	*
$\eta_{cc}$	Calculated from $\bar{m}_c(m_c)$ and $\alpha_s$	[13]	-	*
$\eta_{ct}$	$0.47 \pm 0.04$	[12]	-	*
$\eta_{tt}$	$0.5765 \pm 0.0065$	[12, 13]	-	*
$\eta_B(\overline{\text{MS}})$	$0.551 \pm 0.007$	[14]	-	*
$f_{B_d}$	$(191 \pm 27) \text{ MeV}$	[1]	*	-
$B_d$	$1.37 \pm 0.14$	[1]	*	-
$\xi^{(a)}$	$1.24 \pm 0.04 \pm 0.06$	[1]	*	*

<sup>(a)</sup> anticorrelated theory error with  $f_{B_d} \sqrt{B_d}$ .

Table 1: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the ‘‘GS’’ column by an asterisk; measurements or parameters that have systematic theoretical errors are marked in the ‘‘TH’’ column by an asterisk. Upper part: experimental determinations of the CKM matrix elements. Middle upper part: CP-violation and mixing observables. Middle lower part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

Observable	central $\pm \pm CL \equiv 1\sigma$	$\pm \pm CL \equiv 2\sigma$	$\pm \pm CL \equiv 3\sigma$
$\lambda$	$0.22717^{+0.00100}_{-0.00101}$	$+0.00200$ $-0.00202$	$+0.00300$ $-0.00304$
$A$	$0.806^{+0.014}_{-0.014}$	$+0.029$ $-0.028$	$+0.044$ $-0.042$
$\bar{\rho}$	$0.195^{+0.022}_{-0.055}$	$+0.044$ $-0.110$	$+0.067$ $-0.138$
$\bar{\eta}$	$0.326^{+0.027}_{-0.015}$	$+0.053$ $-0.030$	$+0.074$ $-0.046$
$J$ [ $10^{-5}$ ]	$2.91^{+0.25}_{-0.14}$	$+0.51$ $-0.28$	$+0.72$ $-0.42$
$\sin 2(\alpha)$	$-0.31^{+0.32}_{-0.13}$	$+0.62$ $-0.25$	$+0.75$ $-0.38$
$\sin 2(\alpha)$ (meas. not in fit)	$-0.34^{+0.25}_{-0.14}$	$+0.72$ $-0.27$	$+0.88$ $-0.38$
$\sin 2(\beta)$	$0.695^{+0.018}_{-0.016}$	$+0.041$ $-0.033$	$+0.066$ $-0.050$
$\sin 2(\beta)$ (meas. not in fit)	$0.823^{+0.016}_{-0.085}$	$+0.031$ $-0.146$	$+0.045$ $-0.172$
$\alpha$ (deg)	$99.0^{+4.0}_{-9.4}$	$+8.0$ $-17.9$	$+12.6$ $-22.0$
$\alpha$ (deg) (meas. not in fit)	$100.0^{+4.5}_{-7.3}$	$+8.9$ $-21.2$	$+13.1$ $-26.2$
$\alpha$ (deg) (dir. meas.)	$92.6^{+10.7}_{-9.3}$	$+27.1$ $-15.7$	$+35.8$ $-20.6$
$\beta$ (deg)	$22.03^{+0.72}_{-0.62}$	$+1.69$ $-1.27$	$+2.76$ $-1.93$
$\beta$ (deg) (meas. not in fit)	$27.70^{+0.78}_{-3.94}$	$+1.59$ $-6.41$	$+2.41$ $-7.40$
$\beta$ (deg) (dir. meas.)	$21.23^{+1.03}_{-0.99}$	$+2.09$ $-1.96$	$+3.20$ $-2.90$
$\gamma \simeq \delta$ (deg)	$59.0^{+9.2}_{-3.7}$	$+18.0$ $-7.3$	$+22.3$ $-11.3$
$\gamma \simeq \delta$ (deg) (meas. not in fit)	$59.0^{+9.3}_{-3.8}$	$+18.1$ $-7.4$	$+22.3$ $-11.5$
$\gamma \simeq \delta$ (deg) (dir. meas.)	$60^{+38}_{-24}$	$+62$ $-39$	$+90$ $-51$
$\sin \theta_{12}$	$0.22715^{+0.00102}_{-0.00100}$	$+0.00202$ $-0.00201$	$+0.00301$ $-0.00302$
$\sin \theta_{13}$ [ $10^{-3}$ ]	$3.682^{+0.107}_{-0.078}$	$+0.264$ $-0.158$	$+0.434$ $-0.238$
$\sin \theta_{23}$ [ $10^{-3}$ ]	$41.61^{+0.62}_{-0.63}$	$+1.24$ $-1.25$	$+1.88$ $-1.87$
$R_u$	$0.3798^{+0.0107}_{-0.0090}$	$+0.0256$ $-0.0182$	$+0.0418$ $-0.0274$
$R_t$	$0.868^{+0.060}_{-0.025}$	$+0.118$ $-0.049$	$+0.146$ $-0.075$
$\Delta m_d$ ( $\text{ps}^{-1}$ ) (meas. not in fit)	$0.52^{+0.14}_{-0.10}$	$+0.23$ $-0.22$	$+0.31$ $-0.26$
$\Delta m_s$ ( $\text{ps}^{-1}$ ) (meas. not in fit)	$18.9^{+5.7}_{-2.8}$	$+12.1$ $-5.5$	$+16.1$ $-7.2$
$\epsilon_K$ [ $10^{-3}$ ] (meas. not in fit)	$1.90^{+1.14}_{-0.40}$	$+1.59$ $-0.55$	$+1.95$ $-0.69$
$\xi_{SU(3)}^{\Delta m_{s,d}}$ (lattice value not in fit)	$1.158^{+0.096}_{-0.064}$	$+0.198$ $-0.168$	$+0.260$ $-0.211$
$f_{B_d}$ (MeV) (lattice value not in fit)	$187^{+11}_{-11}$	$+23$ $-21$	$+37$ $-30$
$B_K$ (lattice value not in fit)	$0.87^{+0.14}_{-0.23}$	$+0.24$ $-0.31$	$+0.34$ $-0.35$
$m_c$ ( $\text{GeV}/c^2$ ) (meas. not in fit)	$0.97^{+0.85}_{-0.48}$	$+0.98$ $-0.48$	$+1.12$ $-0.48$
$m_t$ ( $\text{GeV}/c^2$ ) (meas. not in fit)	$138^{+26}_{-16}$	$+62$ $-29$	$+90$ $-39$

Table 2: *Fit results.*

Observable	central $\pm \pm CL \equiv 1\sigma$	$\pm \pm CL \equiv 2\sigma$	$\pm \pm CL \equiv 3\sigma$
$\mathcal{B}(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})$ [ $10^{-11}$ ]	$2.38^{+0.53}_{-0.36}$	$+1.04$ $-0.57$	$+1.49$ $-0.76$
$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ [ $10^{-11}$ ]	$7.4^{+2.0}_{-2.0}$	$+2.9$ $-2.4$	$+3.5$ $-2.6$
$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\mu)$ [ $10^{-5}$ ]	$8.7^{+1.3}_{-2.0}$	$+2.8$ $-2.9$	$+4.6$ $-3.5$
$\mathcal{B}(B^+ \rightarrow \mu^+ \nu_\mu)$ [ $10^{-7}$ ]	$4.15^{+0.56}_{-0.64}$	$+1.25$ $-1.30$	$+2.09$ $-1.66$
$ V_{ud} $	$0.97385^{+0.00024}_{-0.00023}$	$+0.00047$ $-0.00047$	$+0.00071$ $-0.00070$
$ V_{us} $	$0.22715^{+0.00101}_{-0.00100}$	$+0.00201$ $-0.00201$	$+0.00301$ $-0.00303$
$ V_{ub} $ [ $10^{-3}$ ]	$3.683^{+0.106}_{-0.079}$	$+0.263$ $-0.158$	$+0.434$ $-0.239$
$ V_{ub} $ [ $10^{-3}$ ] (meas. not in fit)	$3.59^{+0.17}_{-0.18}$	$+0.34$ $-0.36$	$+0.51$ $-0.52$
$ V_{cd} $	$0.22703^{+0.00102}_{-0.00100}$	$+0.00202$ $-0.00201$	$+0.00301$ $-0.00303$
$ V_{cs} $	$0.97299^{+0.00024}_{-0.00023}$	$+0.00047$ $-0.00047$	$+0.00071$ $-0.00070$
$ V_{cb} $ [ $10^{-3}$ ]	$41.61^{+0.62}_{-0.63}$	$+1.24$ $-1.25$	$+1.88$ $-1.87$
$ V_{cb} $ [ $10^{-3}$ ] (meas. not in fit)	$42.6^{+3.4}_{-1.9}$	$+4.7$ $-3.7$	$+6.0$ $-5.1$
$ V_{td} $ [ $10^{-3}$ ]	$8.20^{+0.59}_{-0.27}$	$+1.18$ $-0.53$	$+1.48$ $-0.78$
$ V_{ts} $ [ $10^{-3}$ ]	$40.96^{+0.60}_{-0.62}$	$+1.21$ $-1.23$	$+1.84$ $-1.84$
$ V_{tb} $	$0.999127^{+0.000026}_{-0.000026}$	$+0.000052$ $-0.000052$	$+0.000076$ $-0.000080$
$ V_{td}/V_{ts} $	$0.2003^{+0.0146}_{-0.0059}$	$+0.0286$ $-0.0115$	$+0.0354$ $-0.0176$
$ V_{ud}V_{ub}^* $ [ $10^{-3}$ ]	$3.586^{+0.104}_{-0.076}$	$+0.257$ $-0.154$	$+0.423$ $-0.232$
$\arg[V_{ud}V_{ub}^*]$ (deg)	$59.0^{+9.2}_{-3.7}$	$+17.9$ $-7.4$	$+22.2$ $-11.3$
$\arg[-V_{ts}V_{tb}^*]$ (deg)	$1.003^{+0.085}_{-0.046}$	$+0.169$ $-0.092$	$+0.234$ $-0.143$
$ V_{cd}V_{cb}^* $ [ $10^{-3}$ ]	$9.45^{+0.14}_{-0.15}$	$+0.29$ $-0.29$	$+0.44$ $-0.44$
$\arg[-V_{cd}V_{cb}^*]$ (deg)	$0.0323^{+0.0027}_{-0.0016}$	$+0.0056$ $-0.0032$	$+0.0079$ $-0.0048$
$ V_{td}V_{tb}^* $ [ $10^{-3}$ ]	$8.19^{+0.60}_{-0.26}$	$+1.18$ $-0.52$	$+1.49$ $-0.77$
$\arg[V_{td}V_{tb}^*]$ (deg)	$-22.01^{+0.64}_{-0.70}$	$+1.28$ $-1.68$	$+1.94$ $-2.74$
$\text{Re}\lambda_c$	$-0.22091^{+0.00092}_{-0.00093}$	$+0.00186$ $-0.00185$	$+0.00279$ $-0.00276$
$\text{Im}\lambda_c$ [ $10^{-4}$ ]	$-1.312^{+0.063}_{-0.112}$	$+0.126$ $-0.229$	$+0.189$ $-0.324$
$\text{Re}\lambda_t$ [ $10^{-4}$ ]	$-3.09^{+0.13}_{-0.22}$	$+0.26$ $-0.48$	$+0.38$ $-0.65$
$\text{Im}\lambda_t$ [ $10^{-4}$ ]	$1.313^{+0.112}_{-0.063}$	$+0.228$ $-0.127$	$+0.323$ $-0.190$
$\beta_s$ (deg)	$1.005^{+0.086}_{-0.045}$	$+0.169$ $-0.092$	$+0.235$ $-0.143$
$\sin(2\beta_s)$	$0.0351^{+0.0030}_{-0.0016}$	$+0.0059$ $-0.0032$	$+0.0082$ $-0.0050$

Table 3: *Fit results.*