

# UPDATED RESULTS ON THE CKM MATRIX AND THE UNITARITY TRIANGLE

Including results presented up to  
FPCP 06, Vancouver, Canada

*P r e l i m i n a r y*

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The CKMfitter Group

## Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach *Rfit*. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

*CP* VIOLATION AND THE CKM MATRIX:  
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES

By CKMfitter Group

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Parameter	Value $\pm$ Error(s)	Reference	Errors	
			GS	TH
$ V_{ud} $ (nuclei)	$0.97377 \pm 0.00027$	[1]	★	-
$ V_{us} $ ( $K_{\ell 3}$ and $K_{\mu 2}$ )	$0.2257 \pm 0.0021$	[10]	★	-
$ V_{ub} $ (incl.)	$(4.45 \pm 0.23 \pm 0.39) \times 10^{-3}$	[2, 3]	★	★
$ V_{ub} $ (excl.)	$(3.94 \pm 0.28 \pm 0.51) \times 10^{-3}$	[10]	★	★
$ V_{cb} $ (incl.)	$(41.70 \pm 0.70) \times 10^{-3}$	[10]	★	-
$ V_{cb} $ (excl.)	$(41.18 \pm 1.71) \times 10^{-3}$	[2]	★	-
$ \varepsilon_K $	$(2.221 \pm 0.008) \times 10^{-3}$	[5]	★	-
$\Delta m_d$	$(0.507 \pm 0.004) \text{ ps}^{-1}$	[2]	★	-
$\Delta m_s$	Amplitude spectrum+CDF -LogL	[6]	★	-
$\sin(2\beta)_{[c\bar{c}]}$	$0.687 \pm 0.032$	[2]	★	-
$S_{\pi\pi}^{+-}$	$-0.50 \pm 0.12$	[2]	★	-
$C_{\pi\pi}^{+-}$	$-0.37 \pm 0.10$	[2]	★	-
$C_{\pi\pi}^{00}$	$-0.28 \pm 0.39$	[2]	★	-
$\mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis	[2]	★	-
$S_{\rho\rho,L}^{+-}$	$-0.22 \pm 0.22$	[2]	★	-
$C_{\rho\rho,L}^{+-}$	$-0.02 \pm 0.17$	[2]	★	-
$\mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[2]	★	-
$B^0 \rightarrow (\rho\pi)^0 \rightarrow 3\pi$	Time-dependent Dalitz analysis	[7]	★	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to GLW analysis	[2]	★	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to ADS analysis	[2]	★	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	GGSZ Dalitz analyses	[2]	★	-
$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau)$	Experimental likelihoods	[9]	★	-
$\bar{m}_c(m_c)$	$(1.24 \pm 0.037 \pm 0.095) \text{ GeV}$	[4]	★	★
$\bar{m}_t(m_t)$	$(162.3 \pm 2.2) \text{ GeV}$	[8]	★	-
$m_{K^+}$	$(493.677 \pm 0.016) \text{ MeV}$	[10]	-	-
$\Delta m_K$	$(3.4833 \pm 0.0066) \times 10^{-12} \text{ MeV}$	[10]	-	-
$m_{B_d}$	$(5.2794 \pm 0.0005) \text{ GeV}$	[10]	-	-
$m_{B_s}$	$(5.3696 \pm 0.0024) \text{ GeV}$	[10]	-	-
$m_W$	$(80.423 \pm 0.039) \text{ GeV}$	[10]	-	-
$G_F$	$1.16639 \times 10^{-5} \text{ GeV}^{-2}$	[10]	-	-
$f_K$	$(159.8 \pm 1.5) \text{ MeV}$	[10]	-	-
$B_K$	$0.79 \pm 0.04 \pm 0.09$	[1]	★	★
$\alpha_s(m_Z^2)$	$0.1176 \pm 0.0020$	[10]	-	★
$\eta_{ct}$	$0.47 \pm 0.04$	[11]	-	★
$\eta_{tt}$	$0.5765 \pm 0.0065$	[11, 12]	-	★
$\eta_B(\overline{\text{MS}})$	$0.551 \pm 0.007$	[13]	-	★
$f_{B_d}$	$(191 \pm 27) \text{ MeV}$	[1]	★	-
$B_d$	$1.37 \pm 0.14$	[1]	★	-
$\xi^{(a)}$	$1.24 \pm 0.04 \pm 0.06$	[1]	★	★

<sup>(a)</sup> anticorrelated theory error with  $f_{B_d} \sqrt{B_d}$ .

Table 1: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the “GS” column by an asterisk; measurements or parameters that have systematic theoretical errors are marked in the “TH” column by an asterisk. Upper part: experimental determinations of the CKM matrix elements. Middle upper part: CP-violation and mixing observables. Middle lower part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

Observable	central $\pm$ CL $\equiv$ $1\sigma$	$\pm$ CL $\equiv$ $2\sigma$	$\pm$ CL $\equiv$ $3\sigma$
$\lambda$	$0.2272^{+0.0010}_{-0.0010}$	$+0.0020$ $-0.0020$	$+0.0030$ $-0.0030$
$A$	$0.809^{+0.014}_{-0.014}$	$+0.029$ $-0.028$	$+0.044$ $-0.042$
$\bar{\rho}$	$0.197^{+0.026}_{-0.030}$	$+0.050$ $-0.087$	$+0.074$ $-0.133$
$\bar{\eta}$	$0.339^{+0.019}_{-0.018}$	$+0.047$ $-0.037$	$+0.075$ $-0.057$
$J$ [ $10^{-5}$ ]	$3.05^{+0.18}_{-0.18}$	$+0.45$ $-0.36$	$+0.69$ $-0.54$
$\sin(2\alpha)$	$-0.25^{+0.17}_{-0.15}$	$+0.49$ $-0.28$	$+0.71$ $-0.42$
$\sin(2\alpha)$ (meas. not in fit)	$-0.23^{+0.55}_{-0.16}$	$+0.72$ $-0.32$	$+0.83$ $-0.45$
$\sin(2\beta)$	$0.716^{+0.024}_{-0.024}$	$+0.048$ $-0.049$	$+0.074$ $-0.075$
$\sin(2\beta)$ (meas. not in fit)	$0.752^{+0.057}_{-0.035}$	$+0.105$ $-0.073$	$+0.135$ $-0.112$
$\alpha$ (deg)	$97.3^{+4.5}_{-5.0}$	$+8.7$ $-14.0$	$+13.7$ $-20.7$
$\alpha$ (deg) (meas. not in fit)	$96.5^{+4.9}_{-16.0}$	$+9.9$ $-21.2$	$+14.6$ $-25.3$
$\alpha$ (deg) (dir. meas.)	$100.2^{+15.0}_{-8.8}$	$+22.7$ $-18.2$	$+32.0$ $-28.1$
$\beta$ (deg)	$22.86^{+1.00}_{-1.00}$	$+2.03$ $-1.97$	$+3.22$ $-2.93$
$\beta$ (deg) (meas. not in fit)	$24.4^{+2.6}_{-1.5}$	$+5.1$ $-3.0$	$+6.9$ $-4.5$
$\beta$ (deg) (dir. meas.)	$21.7^{+1.3}_{-1.2}$	$+2.6$ $-2.4$	$+4.1$ $-3.6$
$\gamma \simeq \delta$ (deg)	$59.8^{+4.9}_{-4.1}$	$+13.9$ $-7.9$	$+20.8$ $-12.1$
$\gamma \simeq \delta$ (deg) (meas. not in fit)	$59.8^{+4.9}_{-4.2}$	$+14.1$ $-8.0$	$+21.0$ $-12.3$
$\gamma \simeq \delta$ (deg) (dir. meas.)	$63^{+35}_{-25}$	$+62$ $-40$	$+100$ $-54$
$\beta_s$ (deg)	$1.045^{+0.061}_{-0.057}$	$+0.151$ $-0.114$	$+0.238$ $-0.177$
$\sin(2\beta_s)$	$0.0365^{+0.0021}_{-0.0020}$	$+0.0053$ $-0.0040$	$+0.0083$ $-0.0062$
$\sin \theta_{12}$	$0.2272^{+0.0010}_{-0.0010}$	$+0.0020$ $-0.0020$	$+0.0030$ $-0.0030$
$\sin \theta_{13}$ [ $10^{-3}$ ]	$3.82^{+0.15}_{-0.15}$	$+0.31$ $-0.30$	$+0.49$ $-0.44$
$\sin \theta_{23}$ [ $10^{-3}$ ]	$41.78^{+0.63}_{-0.63}$	$+1.26$ $-1.26$	$+1.90$ $-1.89$
$R_u$	$0.391^{+0.015}_{-0.015}$	$+0.031$ $-0.029$	$+0.049$ $-0.044$
$R_t$	$0.872^{+0.033}_{-0.028}$	$+0.095$ $-0.054$	$+0.143$ $-0.082$
$\Delta m_d$ ( $\text{ps}^{-1}$ ) (meas. not in fit)	$0.394^{+0.097}_{-0.097}$	$+0.219$ $-0.132$	$+0.361$ $-0.162$
$\Delta m_s$ ( $\text{ps}^{-1}$ )	$17.34^{+0.49}_{-0.20}$	$+0.65$ $-0.35$	$+0.78$ $-0.49$
$\Delta m_s$ ( $\text{ps}^{-1}$ ) (meas. not in fit)	$21.7^{+5.9}_{-4.2}$	$+9.7$ $-6.8$	$+13.1$ $-9.1$
$\epsilon_K$ [ $10^{-3}$ ] (meas. not in fit)	$2.46^{+0.63}_{-0.88}$	$+1.05$ $-1.05$	$+1.50$ $-1.20$
$f_{B_d}$ (MeV) (lattice value not in fit)	$183^{+10}_{-10}$	$+21$ $-20$	$+34$ $-28$
$\xi_{SU(3)}^{\Delta m_{d,s}}$ (lattice value not in fit)	$1.061^{+0.122}_{-0.047}$	$+0.213$ $-0.083$	$+0.324$ $-0.119$
$B_K$ (lattice value not in fit)	$0.722^{+0.251}_{-0.084}$	$+0.348$ $-0.157$	$+0.461$ $-0.216$
$m_c$ ( $\text{GeV}/c^2$ ) (meas. not in fit)	$0.81^{+0.93}_{-0.36}$	$+1.08$ $-0.36$	$+1.23$ $-0.81$
$m_t$ ( $\text{GeV}/c^2$ ) (meas. not in fit)	$150^{+27}_{-21}$	$+57$ $-35$	$+79$ $-46$

Table 2: *Fit results.*

Observable	central $\pm$ CL $\equiv$ $1\sigma$	$\pm$ CL $\equiv$ $2\sigma$	$\pm$ CL $\equiv$ $3\sigma$
$\mathcal{B}(B^+ \rightarrow \tau^+ \nu_\mu)$ [ $10^{-5}$ ]	$9.6^{+1.5}_{-1.5}$	$+3.3$ $-2.9$	$+5.4$ $-4.0$
$\mathcal{B}(B^+ \rightarrow \mu^+ \nu_\mu)$ [ $10^{-7}$ ]	$4.32^{+0.58}_{-0.57}$	$+1.27$ $-1.12$	$+2.05$ $-1.62$
$\mathcal{B}(K_L^0 \rightarrow \pi^0 \nu \bar{\nu})$ [ $10^{-11}$ ]	$2.58^{+0.48}_{-0.40}$	$+1.01$ $-0.68$	$+1.53$ $-0.93$
$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ [ $10^{-11}$ ]	$7.5^{+1.8}_{-2.0}$	$+2.5$ $-2.4$	$+3.2$ $-2.7$
$ V_{ud} $	$0.97383^{+0.00024}_{-0.00023}$	$+0.00047$ $-0.00047$	$+0.00071$ $-0.00071$
$ V_{us} $	$0.2272^{+0.0010}_{-0.0010}$	$+0.0020$ $-0.0020$	$+0.0030$ $-0.0030$
$ V_{ub} $ [ $10^{-3}$ ]	$3.82^{+0.15}_{-0.15}$	$+0.31$ $-0.29$	$+0.49$ $-0.44$
$ V_{ub} $ [ $10^{-3}$ ] (meas. not in fit)	$3.64^{+0.19}_{-0.18}$	$+0.39$ $-0.36$	$+0.60$ $-0.55$
$ V_{cd} $	$0.22712^{+0.00099}_{-0.00103}$	$+0.00199$ $-0.00205$	$+0.00300$ $-0.00307$
$ V_{cs} $	$0.97297^{+0.00024}_{-0.00023}$	$+0.00048$ $-0.00047$	$+0.00071$ $-0.00071$
$ V_{cb} $ [ $10^{-3}$ ]	$41.79^{+0.63}_{-0.63}$	$+1.26$ $-1.27$	$+1.89$ $-1.90$
$ V_{cb} $ [ $10^{-3}$ ] (meas. not in fit)	$44.9^{+1.2}_{-2.8}$	$+2.4$ $-5.7$	$+3.8$ $-7.7$
$ V_{td} $ [ $10^{-3}$ ]	$8.28^{+0.33}_{-0.29}$	$+0.92$ $-0.57$	$+1.38$ $-0.86$
$ V_{ts} $ [ $10^{-3}$ ]	$41.13^{+0.63}_{-0.62}$	$+1.25$ $-1.24$	$+1.87$ $-1.86$
$ V_{tb} $	$0.999119^{+0.000026}_{-0.000027}$	$+0.000052$ $-0.000054$	$+0.000078$ $-0.000082$
$ V_{td}/V_{ts} $	$0.2011^{+0.0081}_{-0.0065}$	$+0.0230$ $-0.0127$	$+0.0345$ $-0.0195$
$ V_{ud}V_{ub}^* $ [ $10^{-3}$ ]	$3.72^{+0.15}_{-0.14}$	$+0.30$ $-0.29$	$+0.48$ $-0.43$
$\arg[V_{ud}V_{ub}^*]$ (deg)	$59.8^{+4.9}_{-4.0}$	$+13.9$ $-7.8$	$+20.9$ $-12.1$
$\arg[-V_{ts}V_{tb}^*]$ (deg)	$1.043^{+0.061}_{-0.057}$	$+0.151$ $-0.114$	$+0.238$ $-0.176$
$ V_{cd}V_{cb}^* $ [ $10^{-3}$ ]	$9.49^{+0.15}_{-0.15}$	$+0.30$ $-0.30$	$+0.45$ $-0.45$
$\arg[-V_{cd}V_{cb}^*]$ (deg)	$0.0339^{+0.0021}_{-0.0020}$	$+0.0050$ $-0.0040$	$+0.0077$ $-0.0060$
$ V_{td}V_{tb}^* $ [ $10^{-3}$ ]	$8.27^{+0.33}_{-0.29}$	$+0.93$ $-0.57$	$+1.38$ $-0.85$
$\arg[V_{td}V_{tb}^*]$ (deg)	$-22.84^{+1.00}_{-0.99}$	$+1.98$ $-2.02$	$+2.93$ $-3.21$
$\text{Re}\lambda_c$	$-0.22098^{+0.00095}_{-0.00091}$	$+0.00188$ $-0.00184$	$+0.00282$ $-0.00275$
$\text{Im}\lambda_c$ [ $10^{-4}$ ]	$-1.377^{+0.080}_{-0.084}$	$+0.161$ $-0.203$	$+0.244$ $-0.310$
$\text{Re}\lambda_t$ [ $10^{-4}$ ]	$-3.11^{+0.13}_{-0.14}$	$+0.26$ $-0.36$	$+0.39$ $-0.57$
$\text{Im}\lambda_t$ [ $10^{-4}$ ]	$1.377^{+0.084}_{-0.080}$	$+0.203$ $-0.161$	$+0.310$ $-0.244$

Table 3: *Fit results.*