

Beyond 1st and 3rd generation unitarity triangle: what can we learn from the others ?

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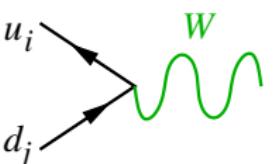
FCCC: flavour-changing charged currents

Electroweak symmetry breaking

- Yukawa interactions not necessarily diagonal in flavour space
- Difference between interaction and mass eigenstate bases

Charged currents at tree level in SM

- diagonal in quark flavour space in interaction basis
- but mixing among generations in mass basis


$$\frac{g}{\sqrt{2}} [\bar{u}_L^i \mathcal{V}_{ij} \gamma^\mu d_L^j W_\mu^+ + \bar{d}_L^j \mathcal{V}_{ij}^* \gamma^\mu u_L^i W_\mu^-]$$

unitary Cabibbo-Kobayashi-Maskawa matrix
(linked to electroweak symmetry breaking)

- Conjugate CKM matrix for CP-conjugate transitions
- CP-violation in quark sector if CKM matrix contains imaginary part
- Depends on the number of physical parameters of the matrix

CKM matrix and CP violation



For two generations, 1 modulus, no phase, no CP violation (Cabibbo)

$$V = \begin{bmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

For three generations, 3 moduli and 1 phase, a unique source of CP violation in quark sector (Kobayashi-Maskawa)

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \simeq \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{bmatrix} + O(\lambda^4)$$

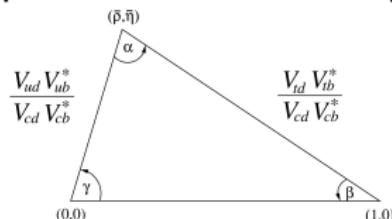
Wolfenstein params exploiting observed hierarchy of matrix elements
⇒ extremely predictive model for CP violation embedded in SM

SM unitarity triangles

Many unitarity relations, e.g., related to 4 neutral mesons (no top)

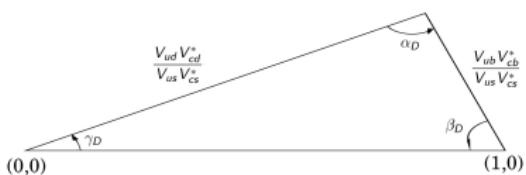
- B_d meson (bd) : $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$ $(\lambda^3, \lambda^3, \lambda^3)$
- B_s meson (bs) : $V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0$ $(\lambda^4, \lambda^2, \lambda^2)$
- K meson (sd) : $V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$ $(\lambda, \lambda, \lambda^5)$
- D meson (cu) : $V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0$ $(\lambda, \lambda, \lambda^5)$

Representation of CKM parameters through rescaled triangles



(small but non squashed)
 B_D -meson triangle (bd)

$$\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} + 1 = 0$$

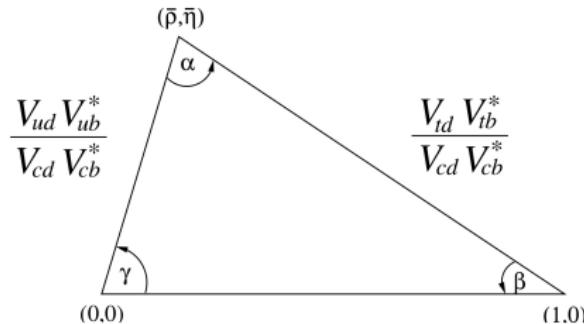


(large but squashed)
 D -meson triangle (cu)

$$\frac{V_{ud} V_{cd}^*}{V_{us} V_{cs}^*} + \frac{V_{ub} V_{cb}^*}{V_{us} V_{cs}^*} + 1 = 0$$

"The" unitarity triangle

In practice, rescaled B_d unitarity triangle often used as representation



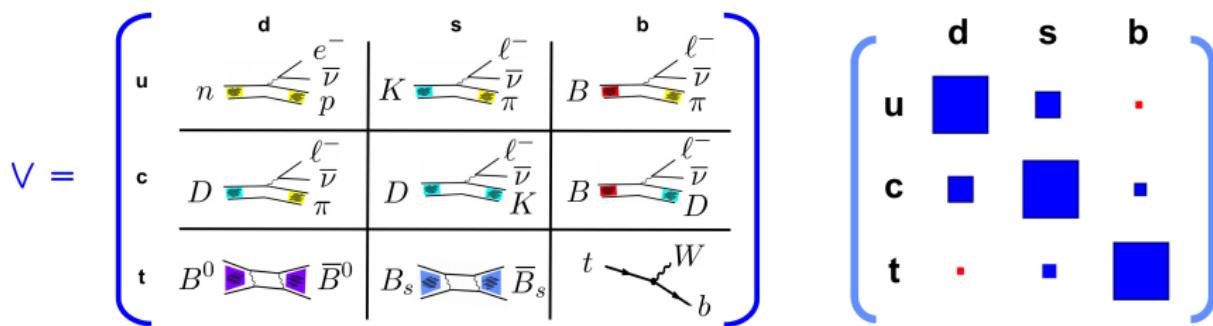
- good representation of CP-violation (small but non-squashed)
- CKM matrix elements involved in interpretation of B decays
- apex yields two of the four Wolfenstein parameters

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad A^2 \lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$

defined in a convention-independent manner

A handle on the CKM matrix

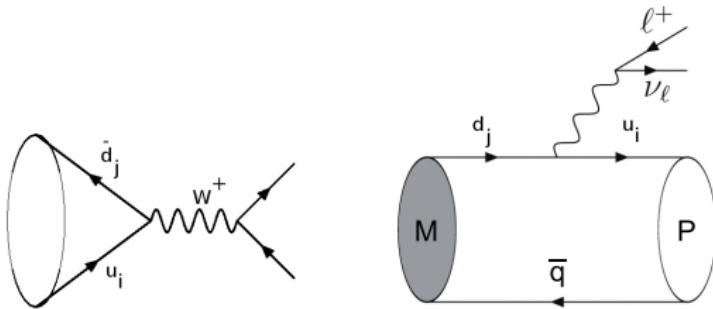
Measurements in terms of hadrons, not of quarks !



- $d \rightarrow u$: Nuclear physics (superallowed β decays)
- $s \rightarrow u$: Kaon physics (KLOE, KTeV, NA62)
- $c \rightarrow d, s$: Charm physics (CLEO-c, Babar, Belle, BESIII)
- $b \rightarrow u, c$ and $t \rightarrow d, s$: B physics (Babar, Belle, CDF, DØ, LHCb)
- $t \rightarrow b$: Top physics (CDF/DØ, ATLAS, CMS)

How to determine the structure of CKM matrix ?

$|V_{ij}|$ from $\Delta F = 1$



- Leptonic, with f_M decay constant

$$B[M \rightarrow \ell \nu_\ell]_{\text{SM}} = \frac{G_F^2 m_M m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_M^2}\right)^2 |V_{q_u q_d}|^2 f_M^2 \tau_M (1 + \delta_{em}^{M\ell 2})$$

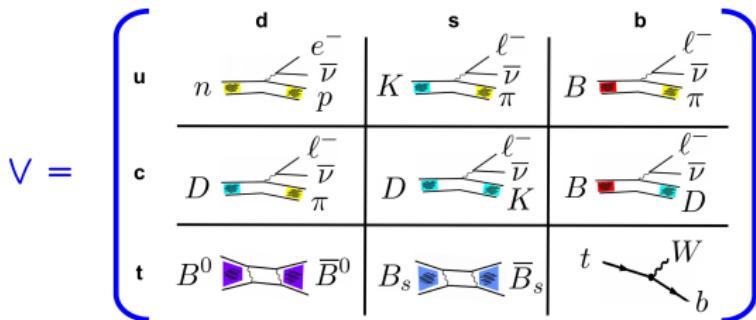
- Semileptonic, with 2 form factors f_+ and f_0

$$\begin{aligned} \frac{d\Gamma(M \rightarrow P \ell \nu)}{dq^2} &= \frac{G_F^2 |V_{q_u q_d}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_H^2} \\ &\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_M^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (m_M^2 - m_P^2)^2 |f_0(q^2)|^2 \right] \end{aligned}$$

- Hadronic quantities, determined from lattice QCD simulations

$$\langle 0 | \bar{q}_u \gamma_\mu \gamma_5 q_d | M \rangle \propto f_M \quad \langle P | \bar{q}_u \gamma_\mu q_d | M \rangle \propto f_+, f_0$$

A few decays of interest



	Leptonic	Semileptonic	Others
$ V_{ud} $	$\pi \rightarrow \ell\nu_\ell, \tau \rightarrow \pi\nu_\tau$	$\pi^+ \rightarrow \pi^0 e^+ \nu_e$	nuclear β decays, n lifetime
$ V_{us} $	$K \rightarrow \ell\nu_\ell, \tau \rightarrow K\nu_\tau$	$K \rightarrow \pi\ell\nu$	inclusive τ decays
$ V_{cd} $	$D^+ \rightarrow \ell\nu_\ell$	$D \rightarrow \pi\ell\nu_\ell$	μ production by ν beams
$ V_{cs} $	$D_s \rightarrow \ell\nu_\ell$	$D \rightarrow K\ell\nu_\ell$	$W \rightarrow c\bar{s}$
$ V_{ub} $	$B \rightarrow \tau\nu$	$B \rightarrow \pi\ell\nu_\ell$	$B \rightarrow X_u \ell\nu_\ell$ (incl)
$ V_{cb} $	$(B_c \rightarrow \tau\nu_\tau)$	$B \rightarrow D(*)\ell\nu$	$B \rightarrow X_c \ell\nu_\ell$ (incl)
$ V_{tb} $	—	—	$t \rightarrow Wb$

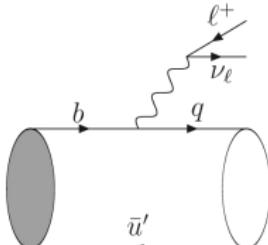
- No direct handle on V_{td} , V_{ts} through tree processes
- Some processes not competitive theo/exp accuracy

$\arg(V_{ij})$ from CP-asymmetries

Take processes conjugate under CP

$$b \rightarrow u : A(\bar{B}^0 \rightarrow \pi^+ \ell^- \bar{\nu}) \propto V_{ub} \times F_{B \rightarrow \pi}$$

$$\bar{b} \rightarrow \bar{u} : A(B^0 \rightarrow \pi^- \ell^+ \nu) \propto V_{ub}^* \times F_{B \rightarrow \pi}$$



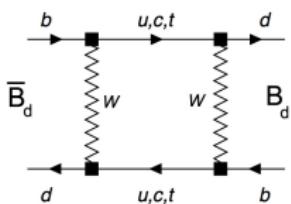
where $F_{B \rightarrow \pi}$ form factor encoding hadronisation of quarks into hadrons

General feature : flavour processes with

- weak part : odd under CP (phase from CKM)
- strong part : even under CP (phase from strong interaction)

- $|V_{ij}|$ via CP-conserving quantity $(|A|^2)$
from rates where hadronic quantities are crucial
- $\arg V_{ij}$ via CP-violating quantity $(\text{Re}(A_1 A_2^*), \text{Im}(A_1 A_2^*))$
from asymmetries where hadronic quantities may cancel out
 \implies CP-viol. from relative phases between conjugate proc.

CKM elements from $\Delta F = 2$

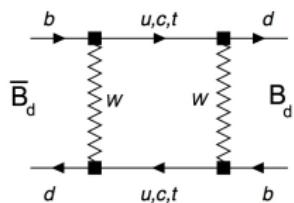


Loops allow $\Delta F = 2$ FCNC

⇒ neutral-meson mixing

$$i \frac{d}{dt} \begin{pmatrix} |M(t)\rangle \\ |\bar{M}(t)\rangle \end{pmatrix} = \left(M - \frac{i}{2}\Gamma \right) \begin{pmatrix} |M(t)\rangle \\ |\bar{M}(t)\rangle \end{pmatrix}$$

CKM elements from $\Delta F = 2$



Loops allow $\Delta F = 2$ FCNC

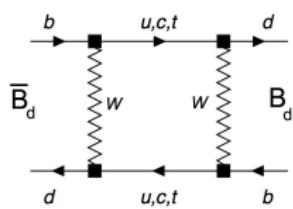
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Diagonalisation: physical $|M_{H,L}\rangle$ of masses $M_{H,L}$, widths $\Gamma_{H,L}$

$$|M_L\rangle = p|M\rangle + q|\bar{M}\rangle, \quad |M_H\rangle = p|M\rangle - q|\bar{M}\rangle \quad |p|^2 + |q|^2 = 1$$

CKM elements from $\Delta F = 2$



Loops allow $\Delta F = 2$ FCNC

\implies neutral-meson mixing

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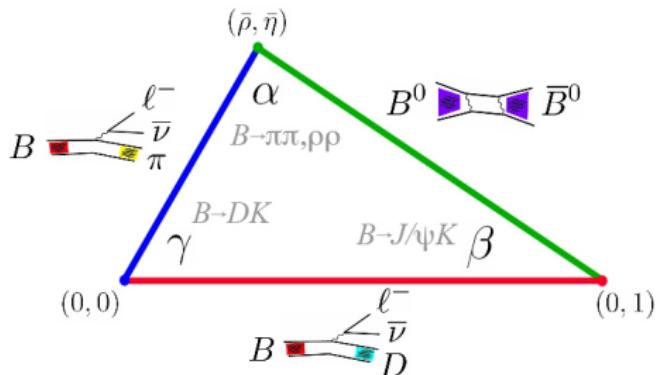
$$|M_L\rangle = p|M\rangle + q|\bar{M}\rangle, \quad |M_H\rangle = p|M\rangle - q|\bar{M}\rangle \quad |p|^2 + |q|^2 = 1$$

For B_d and B_s dominated by top boxes

$$A_{\Delta B=2} \propto (V_{tb}^* V_{tq})^2 \frac{g^4 m_t^2}{16\pi^2 m_W^4} \langle \bar{B}_q | (\bar{b}_L \gamma_\mu d_L)^2 | B_q \rangle + \dots$$

- mass difference Δm_q through hadronic contrib $\langle \bar{B}_q | (\bar{b}_L \gamma_\mu d_L)^2 | B_q \rangle$ (bag parameter B_{B_q})
- mixing involve single weak phase: $q/p = \exp[i \arg[(V_{tb}^* V_{tq})^2]]$
- similar but more complicated for K (charm and top)

A few modes of interest



Exp. uncertainties		(Controlled) th. uncertainties
$B \rightarrow \pi\pi, \rho\rho$	α	$B(b) \rightarrow D(c)\ell\nu$ $ V_{cb} $ vs form factor (OPE)
$B \rightarrow DK$	γ	$B(b) \rightarrow \pi(u)\ell\nu$ $ V_{ub} $ vs form factor (OPE)
		$M \rightarrow \ell\nu(\gamma)$ $ V_{UD} $ vs f_M (decay cst)
$B \rightarrow J/\Psi K_s$	β	$(\bar{\rho}, \bar{\eta})$ vs B_K (bag parameter)
$B_s \rightarrow J/\Psi \phi$	β_s	$B_d \bar{B}_d, B_s \bar{B}_s$ mix $ V_{tb} V_{tq} $ vs $f_B^2 B_B$ (bag param)

- branching ratios of leptonic/semileptonic decays (moduli)
- CP-asymmetries (angles of unitarity triangles(s))
- neutral-meson mixing (product of CKM matrix elements)

Inputs for Summer 19 global fit

CKM
fitter

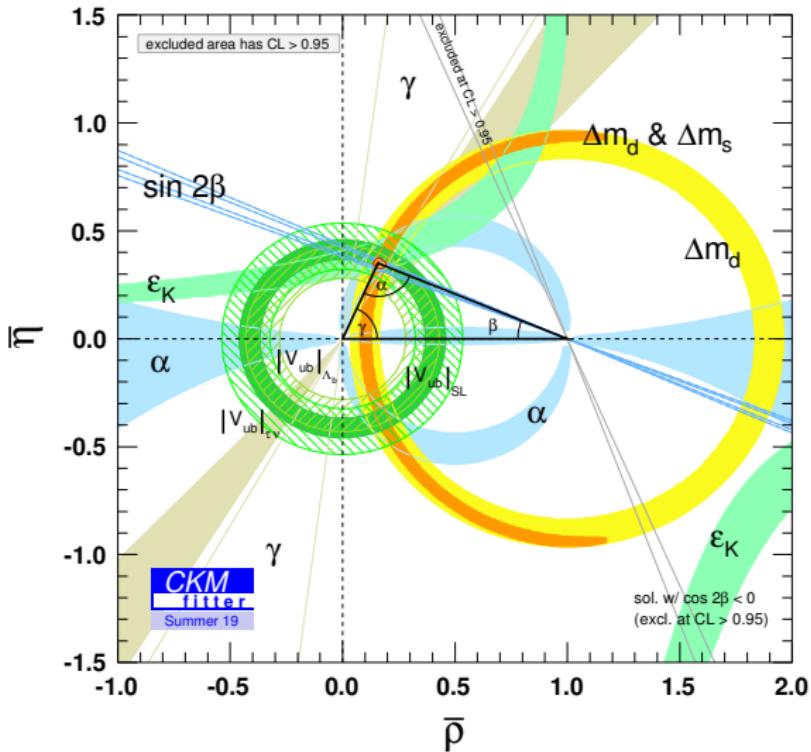
frequentist ($\simeq \chi^2$ minim.) + Rfit scheme for theory uncert.

data = weak \otimes QCD \implies Need for hadronic inputs (mostly lattice)

$ V_{ud} $	superallowed β decays	Towner and Hardy
$ V_{us} $	$K_{\ell 3}$ PDG	$f_+(0) = 0.9681 \pm 0.0014 \pm 0.0022$
	$K \rightarrow \ell\nu, \tau \rightarrow K\nu_\tau$ PDG	$f_K = 155.6 \pm 0.2 \pm 0.6$ MeV
$ V_{us}/V_{ud} $	$K \rightarrow \ell\nu/\pi \rightarrow \ell\nu, \tau \rightarrow K\nu_\tau/\tau \rightarrow \pi\nu_\tau$	$f_K/f_\pi = 1.1973 \pm 0.0008 \pm 0.0014$
ϵ_K	PDG	$\hat{B}_K = 0.7567 \pm 0.0021 \pm 0.0123$
$ V_{cd} $	$D \rightarrow \mu\nu, D \rightarrow \tau\nu, D \rightarrow \pi\ell\nu$	$f_{D_s}/f_D = 1.175 \pm 0.001 \pm 0.004, f_+^{D \rightarrow \pi}(0)$
$ V_{cs} $	$D_s \rightarrow \mu\nu, D_s \rightarrow \tau\nu, D \rightarrow K\ell\nu$	$f_{D_s} = 249.2 \pm 0.3 \pm 0.7$ MeV, $f_+^{D \rightarrow K}(0)$
$ V_{ub} $	inclusive and exclusive B semileptonic	$ V_{ub} \cdot 10^3 = 3.91 \pm 0.08 \pm 0.21$
$ V_{cb} $	inclusive and exclusive B semileptonic	$ V_{cb} \cdot 10^3 = 41.1 \pm 0.3 \pm 0.5$
$B \rightarrow \tau\nu$	$(1.09 \pm 0.24) \cdot 10^{-4}$	$f_{B_s}/f_{B_d} = 1.205 \pm 0.003 \pm 0.006$
		$f_{B_s} = 228.8 \pm 0.7 \pm 1.9$ MeV
$ V_{ub}/V_{cb} $	Λ_b semileptonic decays	integrals of Λ_b form factors
Δm_d	last WA B_d - \bar{B}_d mixing	$B_{B_s}/B_{B_d} = 1.007 \pm 0.013 \pm 0.014$
Δm_s	last WA B_s - \bar{B}_s mixing	$B_{B_s} = 1.327 \pm 0.016 \pm 0.030$
β	last WA $(c\bar{c})$ $K^{(*)}$	no penguin pollution
α	last WA $\pi\pi, \rho\pi, \rho\rho$	isospin
γ	last WA $B \rightarrow D^{(*)}K^{(*)}$	GLW/ADS/GGSZ

as well as inputs on $m_t, m_c, \alpha_s(M_Z)$

The current status of CKM



$|V_{ud}|, |V_{us}|, |V_{cb}|, |V_{ub}|_{SL}$

$B \rightarrow \tau\nu, |V_{ub}|_{\Lambda_b}$

$\Delta m_d, \Delta m_s, \epsilon_K$

$\alpha, \sin 2\beta, \gamma$

$$A = 0.823 {}^{+0.011}_{-0.023}$$

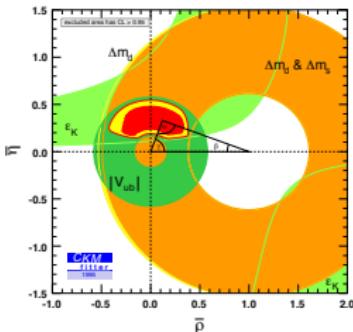
$$\lambda = 0.22484 {}^{+0.00025}_{-0.00006}$$

$$\bar{\rho} = 0.157 {}^{+0.010}_{-0.006}$$

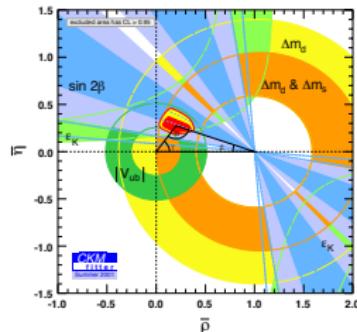
$$\bar{\eta} = 0.350 {}^{+0.008}_{-0.007}$$

(68% CL)

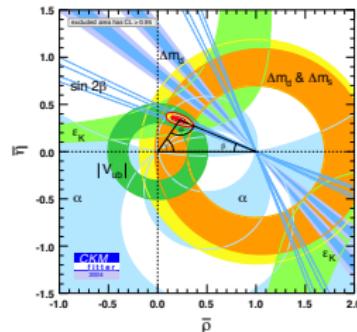
Two decades of CKM



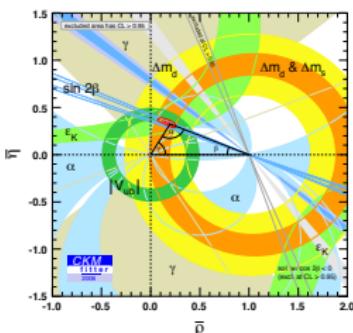
1995



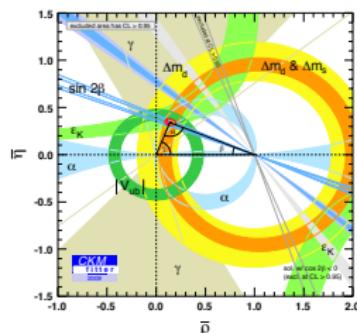
2001



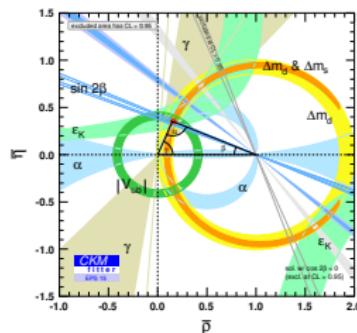
2004



2006

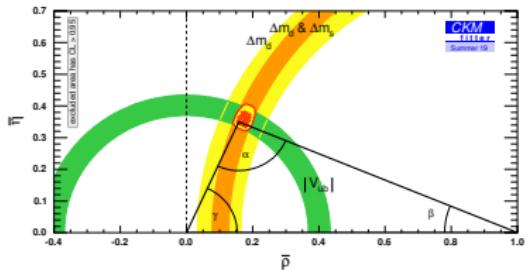


2009

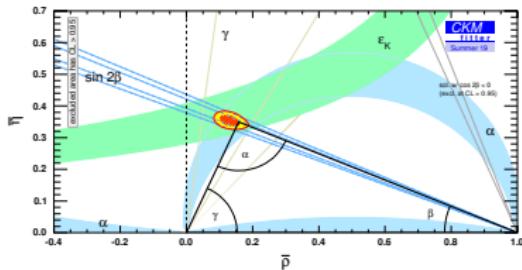


2015

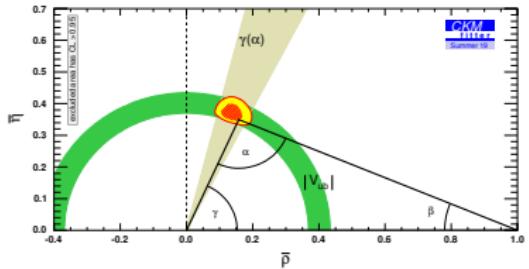
Consistency of the KM mechanism



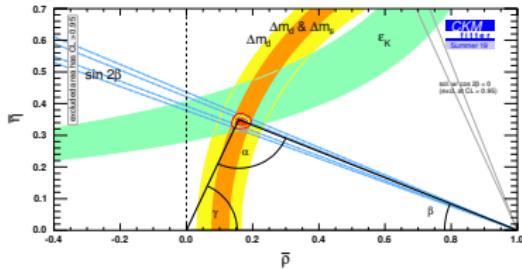
CP-allowed only



CP-violating only



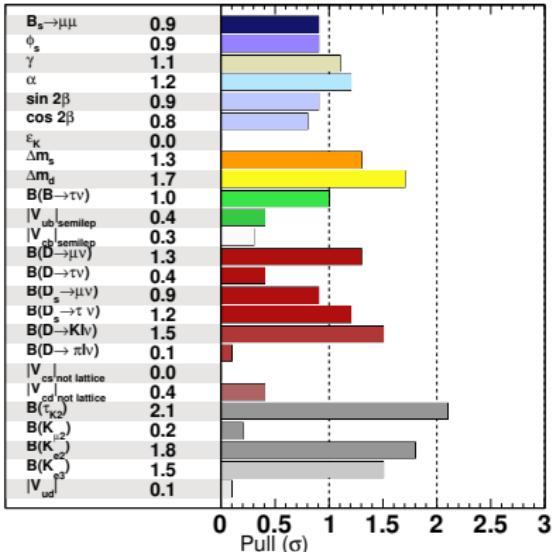
Tree only



Loop only

Validity of Kobayashi-Maskawa picture of *CP* violation

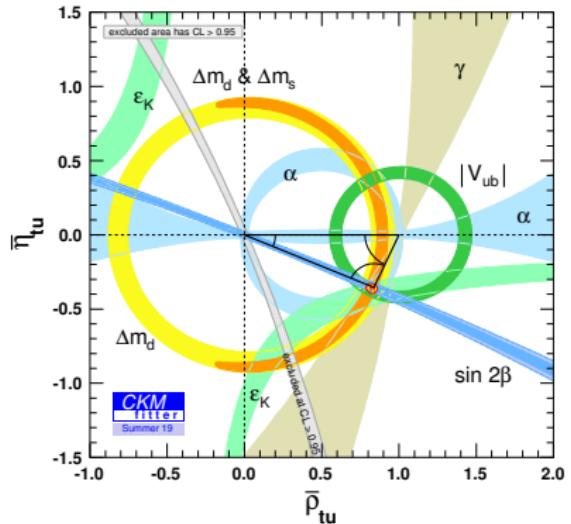
Pulls



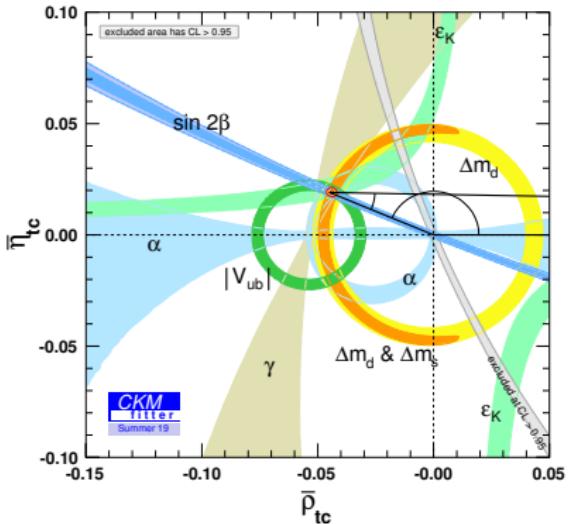
- Pulls for various observables (included in the fit or not)
- For 1D, pull obs = $\sqrt{\chi^2_{\text{min; with obs}} - \chi^2_{\text{min; w/o obs}}}$
- If Gaussian errors, uncorrelated, random vars of mean 0 and variance 1
- Here correlations, and some pulls = 0 due to the Rfit model for syst

No significant deviations from CKM picture

Other triangles: (tu) , (tc)

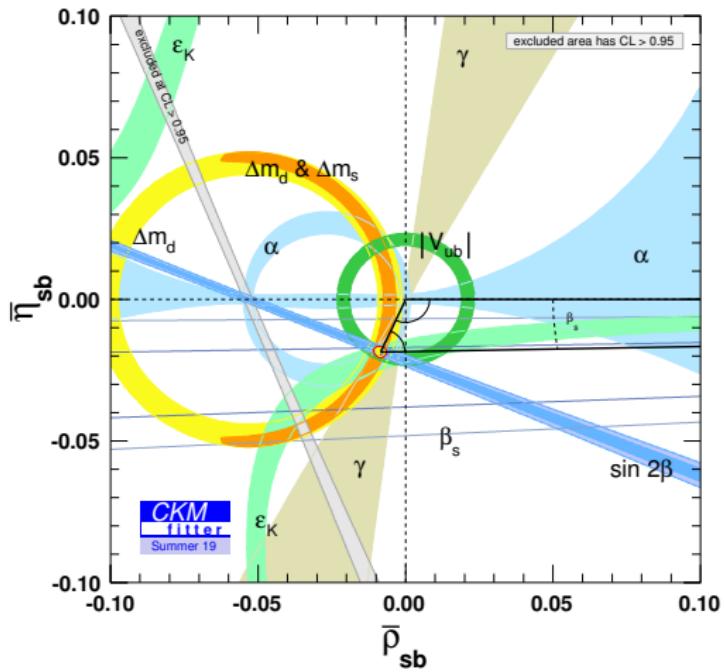


$$\bar{\rho}_{tu} + i\bar{\eta}_{tu} = - \frac{V_{td} V_{ud}^*}{V_{ts} V_{us}^*} (\lambda^3, \lambda^3, \lambda^3)$$



$$\bar{\rho}_{tc} + i\bar{\eta}_{tc} = - \frac{V_{td} V_{cd}^*}{V_{ts} V_{cs}^*} (\lambda^4, \lambda^2, \lambda^2)$$

Other triangles: B_s

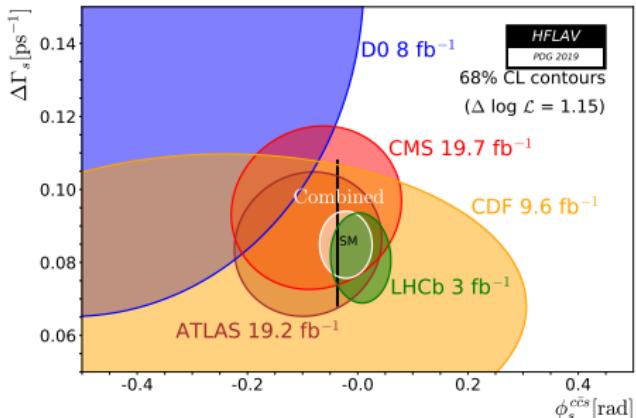


- $\bar{\rho}_{B_s} + i\bar{\eta}_{B_s} = -\frac{V_{us} V_{ub}^*}{V_{cs} V_{cb}^*}$ provides the B_s Unitarity Triangle ($\lambda^4, \lambda^2, \lambda^2$)
- Information on B_s mixing angle β_s from $B_s \rightarrow J/\psi \phi$
- Not relevant for SM determination of CKM parameters, but interesting test of NP

$$\bar{\rho}_{Bs} = -0.00840^{+0.00034}_{-0.00053}$$

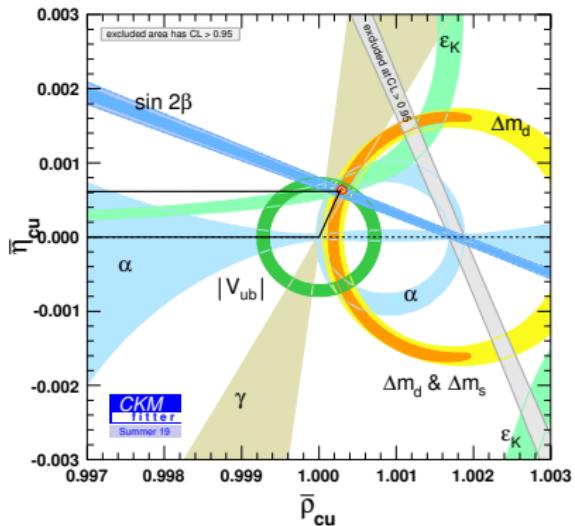
$$\bar{\eta}_{Bs} = -0.01863^{+0.00031}_{-0.00039}$$

CKM and B_s meson



- B_s mixing angle ϕ_s
 - measured through CP-asymmetry in $B_s \rightarrow J/\psi\phi$
 - potentially affected by penguin pollution (different CKM structure)
 - estimated as small, in principle constrained through $SU(3)$ symmetry ($B_s \rightarrow J/\psi\rho$, $B_s \rightarrow J/\psi\bar{K}^*$...)
- [Fleischer et al, Jung, Nierste]
- $\Delta\Gamma_s$
 - Challenging computation through Operator Product Expansion
 - Additional matrix elements compared to Δm_s (lattice QCD)
 - Several contributions at LO + $1/m_b$ corrections
- [Buras, Beneke, Nierste, Lenz]

Other triangles: D



$$\bar{\rho}_{cu} + i\bar{\eta}_{cu} = -\frac{V_{cd} V_{ud}^*}{V_{cs} V_{us}^*} (\lambda, \lambda, \lambda^5)$$

- $\alpha_D = \arg \left(-\frac{V_{ub} V_{cb}^*}{V_{ud} V_{cd}^*} \right) = \arg \left(-\frac{V_{ub} V_{ud}^*}{V_{cb} V_{cd}^*} \right) = -\gamma$
- $\gamma_D = \arg \left(-\frac{V_{ud} V_{cd}^*}{V_{us} V_{cs}^*} \right) = O(\lambda^4)$
- $\beta_D = \arg \left(-\frac{V_{us} V_{cs}^*}{V_{ub} V_{cb}^*} \right) = \pi - \alpha_D - \gamma_D = \pi + \gamma + O(\lambda^4)$

CKM and D and D_s mesons

Measurements difficult to interpret

- CP-violation in D decays
- $D\bar{D}$ mixing

because of

- long-distance contributions from soft gluons/hadronic physics
- non-local matrix elements ($D\bar{D}$ mixing) or meson rescattering ($D \rightarrow \pi\pi, KK$)
- size of $m_c = O(\Lambda_{QCD})$ and CKM hierarchy (light quarks favoured)
- so difficult to separate scales (no expansion for EFT...)

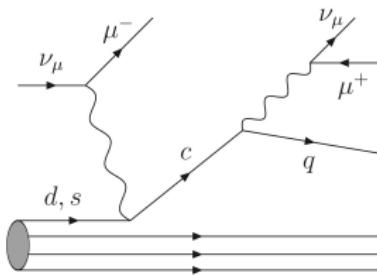
Measurements easy to interpret

- νN scattering and $W \rightarrow cs$ decay
- Leptonic and semileptonic decays

Direct non-lattice constraints on $|V_{cd}|$ and $|V_{cs}|$

- $|V_{cd}|$: deep-inelastic scattering of $\nu, \bar{\nu}$ on nucleons

$$\frac{d^3\sigma(\nu_\mu N \rightarrow \mu^+ \mu^- X)}{d\xi dy dx} = \frac{d^2\sigma(\nu_\mu N \rightarrow cX)}{d\xi dy} D_{c \rightarrow \text{hadron}}(z) B_c(c \rightarrow \mu^+ X)$$



D hadronisation of c , B_c weighted average of cross-sections of charm mesons

$$\frac{d^2\sigma_{LO}(\nu_\mu N \rightarrow cX)}{d\xi dy} \propto [|V_{cd}|^2 d(\xi) + |V_{cs}|^2 s(\xi)]$$

νN vs $\bar{\nu} N$: s cancel and $|V_{cd}|$ up to modelling of d parton distribution

- $|V_{cs}|$: charmed-tagged W decays $W \rightarrow cs$

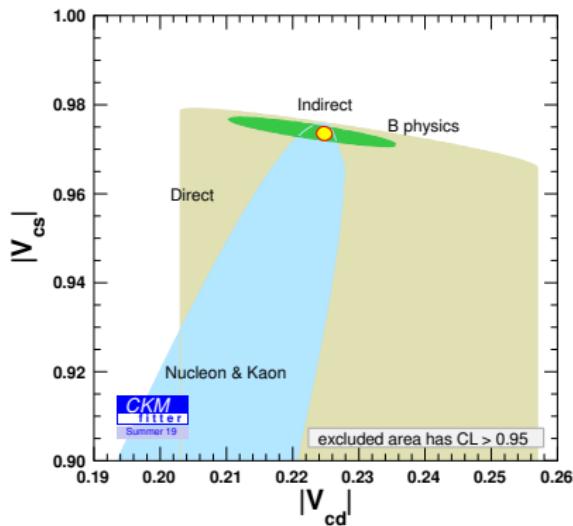
$$|V_{cd}| = 0.230 \pm 0.011$$

$$\sigma(|V_{cd}|)/|V_{cd}| = 5\%$$

$$|V_{cs}| = 0.94^{+0.32}_{-0.26} \pm 0.13$$

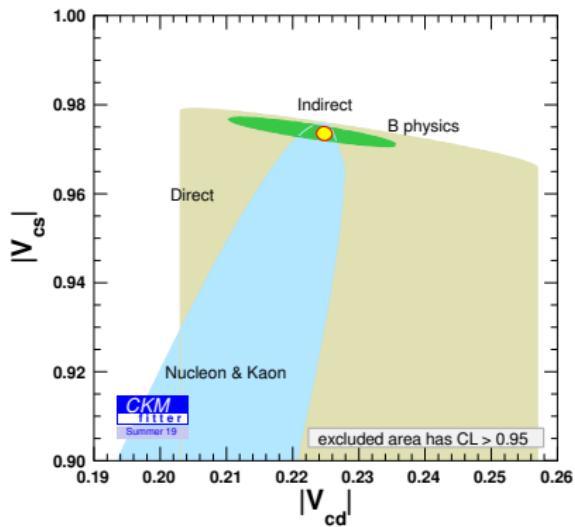
$$\sigma(|V_{cs}|)/|V_{cs}| = 34\% (!)$$

Direct (non-lattice) vs indirect measurements



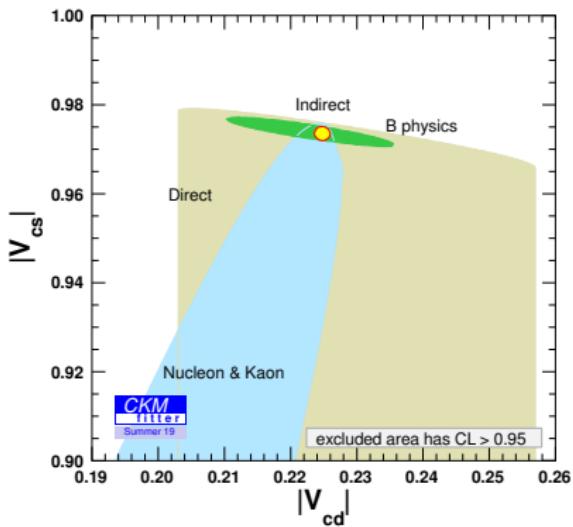
- K and nucleon: not so constraining $V_{ud} \simeq V_{cs}$ and $V_{cd} \simeq V_{us}$ only at first non trivial order in λ (need b -sector to fix the higher orders)

Direct (non-lattice) vs indirect measurements



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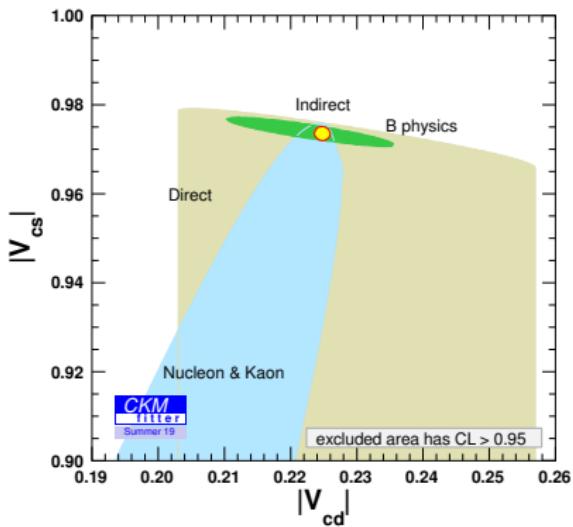
Direct (non-lattice) vs indirect measurements



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- Indirect (combination of the two above): already quite well determined

Direct (non-lattice) vs indirect measurements



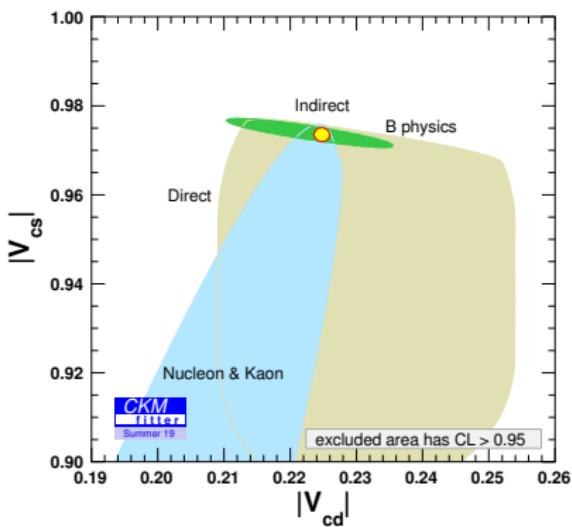
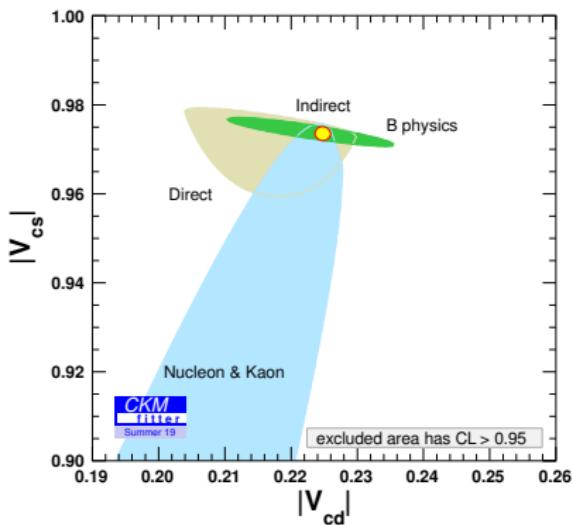
- K and nucleon: not so constraining $V_{ud} \simeq V_{cs}$ and $V_{cd} \simeq V_{us}$ only at first non trivial order in λ (need b -sector to fix the higher orders)
- B alone: rather constraining

- Indirect (combination of the two above): already quite well determined
- Direct (no lattice inputs): poorly known
(ellipse deformed by $|V_{cd}|^2 + |V_{cs}|^2 \leq 1$)

Leptonic and semileptonic decays

	Leptonic		Semileptonic	
Exp	$ V_{cd} $	1.5%	$ V_{cs} $	2.5%
Lattice	0.3%	0.4%	1.2%	0.4%
		3.5%		2.0%

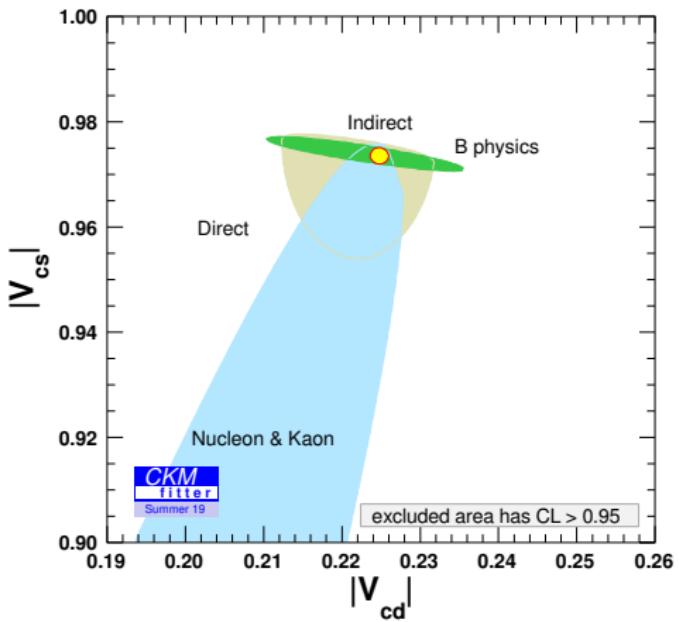
NB: no rad corr included



$$D \rightarrow \tau\nu, D \rightarrow \mu\nu$$
$$D_s \rightarrow \tau\nu, D_s \rightarrow \mu\nu$$

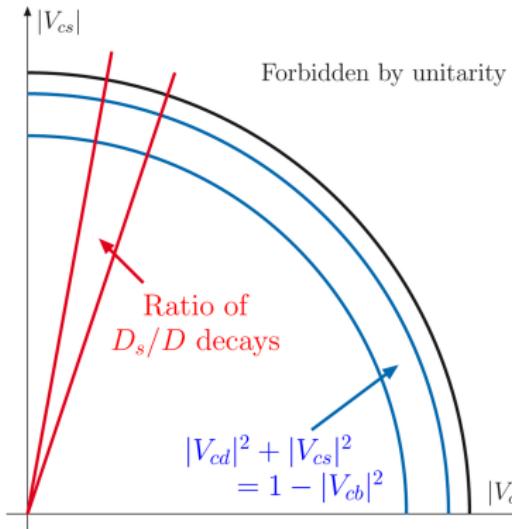
$$D \rightarrow \pi\ell\nu$$
$$D \rightarrow K\ell\nu$$

Altogether



- Direct: $|V_{cd}| = 0.2219^{+0.0039}_{-0.0037}$, $|V_{cs}| = 0.9747^{+0.0012}_{-0.0076}$
- Indirect: $|V_{cd}| = 0.2247^{+0.0003}_{-0.0002}$, $|V_{cs}| = 0.9785^{+0.0005}_{-0.0005}$

The importance of being a ratio



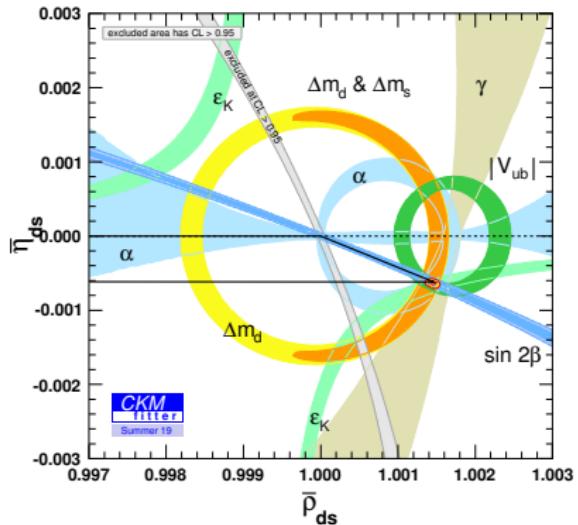
- Unitarity powerful 1D constraint
 $|V_{cd}|^2 + |V_{cs}|^2 = 1 - |V_{cb}|^2$
[even if discrepancy between incl. and excl. determination of $|V_{cb}|$]
- orthogonal constraint from $\frac{|V_{cd}|}{|V_{cs}|}$

$$\frac{\Gamma(D \rightarrow \ell\nu)}{\Gamma(D_s \rightarrow \ell\nu)}, \frac{\Gamma(D \rightarrow \pi e\nu)}{\Gamma(D \rightarrow K e\nu)}$$

- reduced systematics reduced both exp and theo
 f_{D_s}/f_D U-spin breaking, $F_+^\pi(0)/F_+^K(0)\dots$
- vector modes ? for instance, take ratio of $D \rightarrow \pi \ell\nu$ with
 $D_s \rightarrow \phi \ell\nu$, very narrow $s\bar{s}$ state, good control on lattice

Other triangles: K

Measurements difficult to exploit

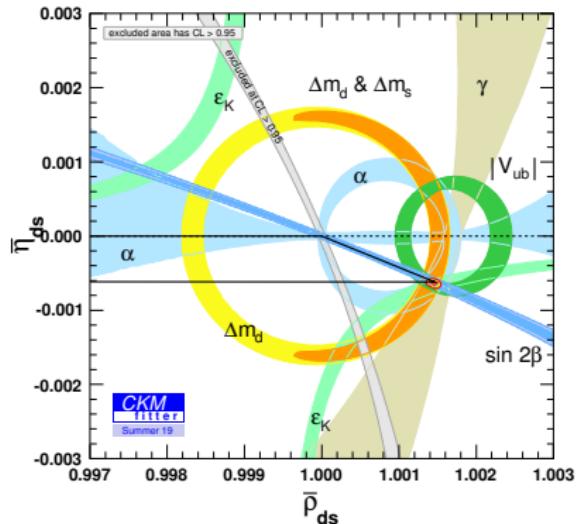


$$\bar{p}_{ds} + i\bar{\eta}_{ds} = -\frac{V_{ud} V_{us}^*}{V_{cd} V_{cs}^*} (\lambda, \lambda, \lambda^5)$$

- Δm_K : large long-distance contribution
- ϵ'/ϵ : measurement of direct CP-violation in $K \rightarrow \pi\pi$, several different long-distance contributions cancelling partially, some lattice estimates hinting at deviation from SM...
- ... but difficulties to get $\pi\pi$ strong phases

[RBC/UKQCD, Buras et al, Pich et al.]

Other triangles: K

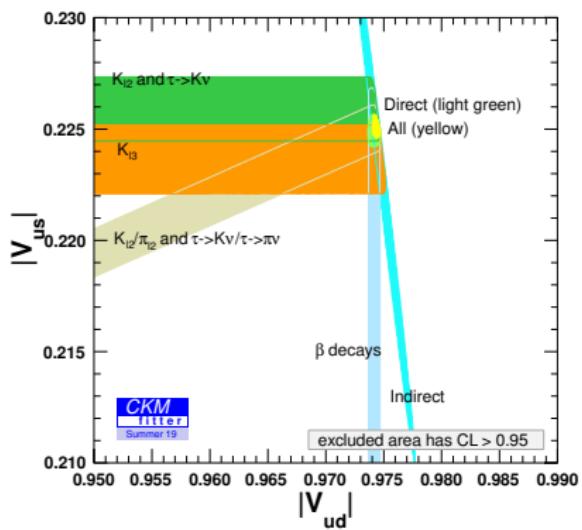


$$\bar{\rho}_{ds} + i\bar{\eta}_{ds} = -\frac{V_{ud} V_{us}^*}{V_{cd} V_{cs}^*} (\lambda, \lambda, \lambda^5)$$

Measurements easy to exploit

- Leptonic decays
- Semileptonic decays
- $K\bar{K}$ mixing from ϵ_K

$|V_{ud}|$ and $|V_{us}|$



- “Direct” (semi- and leptonic) vs “indirect” (other sectors)
- ($|V_{ud}|, |V_{us}|$): nuclear β + leptonic K, π and τ decays
- Same level of accuracy for exp and lattice inputs

	Leptonic	Semilep	
	$ V_{us} $	$ V_{us}/V_{ud} $	$ V_{us} $
Exp	0.1%	0.1%	0.2%
Lattice	0.4%	0.1%	0.3%

- $|V_{ud}|$ from superallowed β decays is 10 times more accurate...

Current controversies on $|V_{ud}|$

$|V_{ud}|$

[Towner, Hardy]

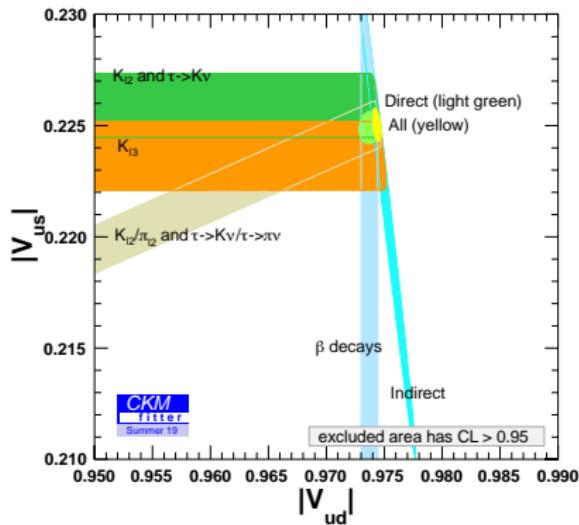
- From superallowed $0^+ \rightarrow 0^+$ nuclear decays
- 15 different transitions considered
- Only vector current of weak interaction concerned
- Good control of radiative and isospin symmetry breaking corrections (claimed to be 0.05 to 0.10%)

Recent re-estimation of the radiative corrections

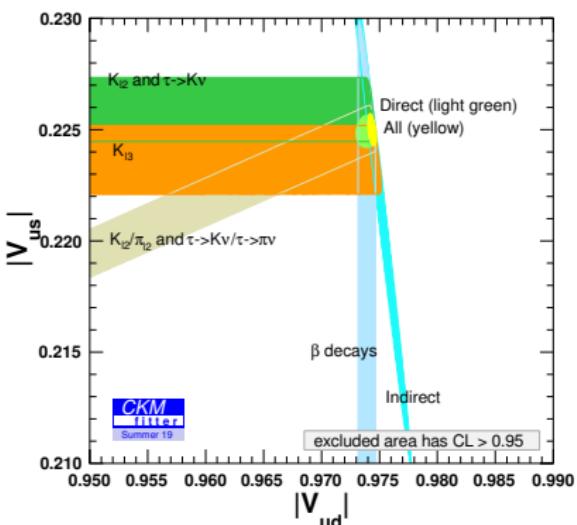
- Towner and Hardy: $|V_{ud}| = 0.97418 \pm 0.00021$
- Gorchtein and Ramsey-Musolf: $|V_{ud}| = 0.97371 \pm 0.00033$
- Czarnecki et al: $|V_{ud}| = 0.97390 \pm 0.00035$

Unitarity of the 1st row and global fit

- Towner and Hardy: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00082^{+0.00084}_{-0.00013}$
- Gorchtein and Ramsey-Musolf:
 $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00198^{+0.00141}_{-0.00012}$
- Czarnecki et al: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00162^{+0.00146}_{-0.00015}$



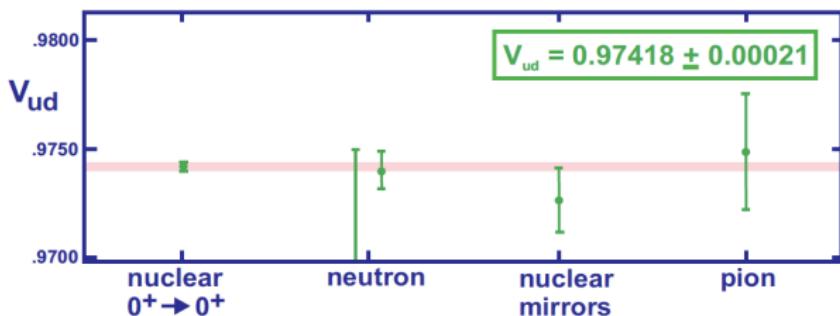
Gorchtein, Ramsey-Musolf



Czarnecki et al

Other sources of information on $|V_{ud}|$

[Towner]



- neutron lifetime (V, A): discrepancy between the various methods of measurements
- nuclear mirror transitions (V, A): β -decay transitions between isobaric analogue states within an isospin doublet (same spin and parity), dominated by experimental uncertainties
- $\pi^+ \rightarrow \pi^0 e^+ \nu_e$ (V): not accurate enough, dominated by experimental uncertainties

$K \rightarrow e\nu$, $K \rightarrow \mu\nu$, $\tau \rightarrow K\nu$ and rad corr

[Marciano-Sirlin, Decker-Finkemeier, Cirigliano-Rosell]

$$B = B_0 \times \text{short-dist. ew corr} \times \text{long-dist. ew corr} \times \text{struct-dep.corr}$$

- Short. dist. expressing W exchanges in terms of G_F [universal]
- Long. dist. using a point-like meson [universal]
- Struct. dep. probing the structure of the meson [process-dep.]

$$B(K \rightarrow \ell\nu) = \frac{G_F^2 |V_{us}|^2}{8\pi} f_K^2 m_K m_\ell^2 \left(1 - \frac{m_\ell^2}{M_K^2}\right)^2 \left(1 + 2\frac{\alpha}{\pi} \log \frac{M_Z}{M_\rho}\right)$$

$$\left(1 + \frac{\alpha}{\pi} F(m_\ell/m_K)\right) (1 + O(\alpha, m_d - m_u))$$

$$B(\tau \rightarrow K\nu_\tau) = \frac{G_F^2 |V_{us}|^2}{16\pi} f_K^2 m_K m_\ell^2 \left(1 - \frac{m_K^2}{M_\tau^2}\right)^2 \left(1 + 2\frac{\alpha}{\pi} \log \frac{M_Z}{M_\tau}\right)$$

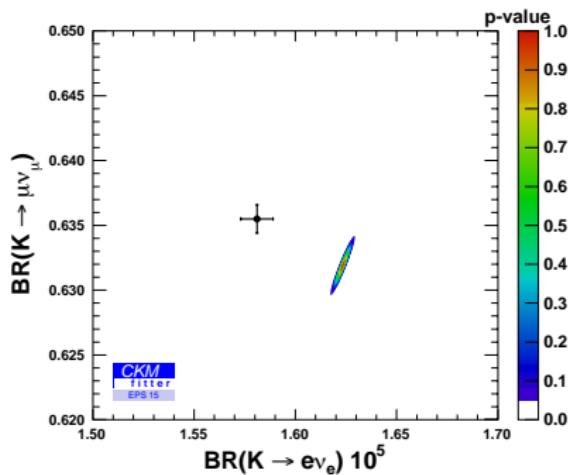
$$\left(1 + \frac{\alpha}{\pi} G(m_K/m_\tau)\right) (1 + O(\alpha, m_d - m_u))$$

NB: First estimates of radiative corrections from lattice QCD ($K\ell\nu/\pi\ell\nu$) in very good agreement with these estimates

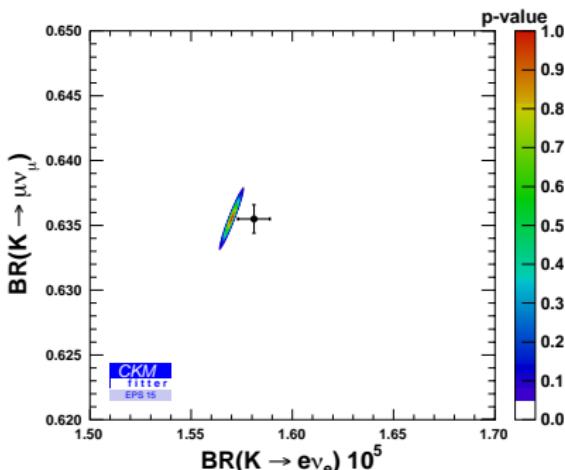
[D. Giusti et al]

The importance of radiative corrections

Comparing the indirect fit results with the measurement for $Br(K \rightarrow \ell\nu)$
⇒ Good test of radiative corrections and lattice QCD !



No radiative corrections



Radiative corrections

NB: No such corrections available (yet) for semileptonic decays

- Measurement of indirect CP-violation in $K \rightarrow \pi\pi$ decays
- Comparing $\pi^+\pi^-$ and $\pi^0\pi^0$
- $|\epsilon_K| = (2.228 \pm 0.011) \times 10^{-3}$

$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \hat{B}_K [\text{Im}[(V_{ts} V_{td}^*)^2] \eta_{tt} S(x_t) + 2\text{Im}[(V_{cs} V_{cd}^* V_{ts} V_{td}^*)] \eta_{ct} S(x_c, x_t) + \text{Im}[(V_{cs} V_{cd}^*)^2] \eta_{cc} S(x_c)]$$

- Inami-Lim $S_0(x_q = m_x^2/m_W^2)$
- C_ϵ normalisation
- κ_ϵ correcting factor for some basic hypotheses of the computation

ϵ_K at NNLO

QCD short-distance corrections computed up to NNLO

- Unitarity used to eliminate one CKM structure
- Known for quite a while for tt , ct , cc , but bad convergence of perturbative series for η_{cc}

[Buras, Jamin, Weisz; Brod, Gorbahn]

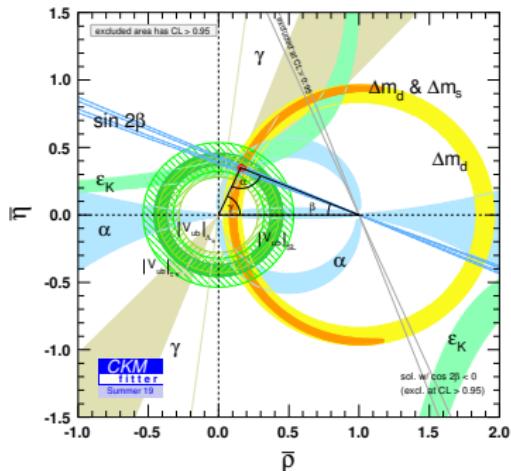
$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \hat{B}_K [\text{Im}[(V_{ts} V_{td}^*)^2] \eta_{tt} S(x_t) + 2\text{Im}[(V_{cs} V_{cd}^* V_{ts} V_{td}^*)] \eta_{ct} S(x_c, x_t) + \text{Im}[(V_{cs} V_{cd}^*)^2] \eta_{cc} S(x_c)]$$

- Recently for tt , ut , uu with far better convergence for ut (and uu negligible)

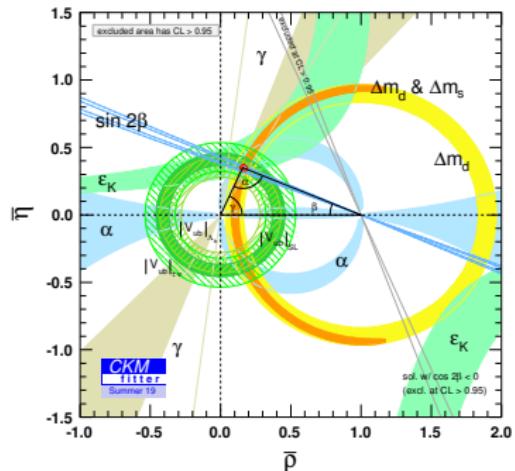
[Brod, Gorbahn]

$$|\epsilon_K| = \kappa_\epsilon C_\epsilon \hat{B}_K [\text{Im}[(V_{ts} V_{td}^*)^2] \eta_{uu} S'(x_t) + 2\text{Im}[(V_{us} V_{ud}^* V_{us} V_{ud}^*)] \eta_{ut} S'(x_c, x_t) + \text{Im}[(V_{us} V_{ud}^*)^2] \eta_{uu} S'(x_c)]$$

ϵ_K at NNLO



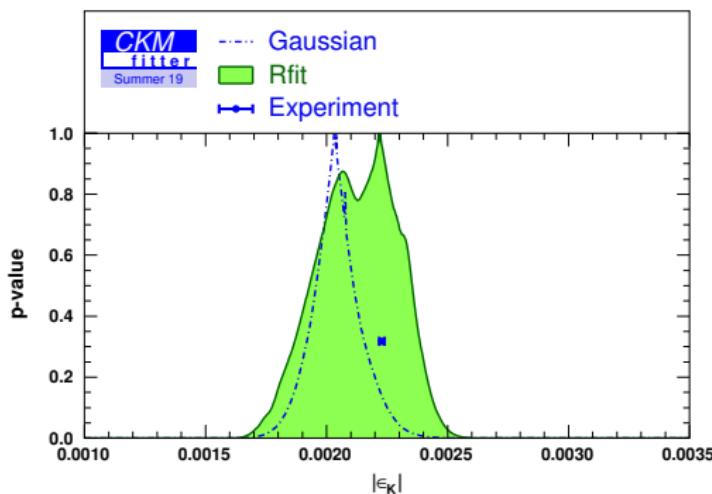
tt, ut, uu



tt, ct, cc

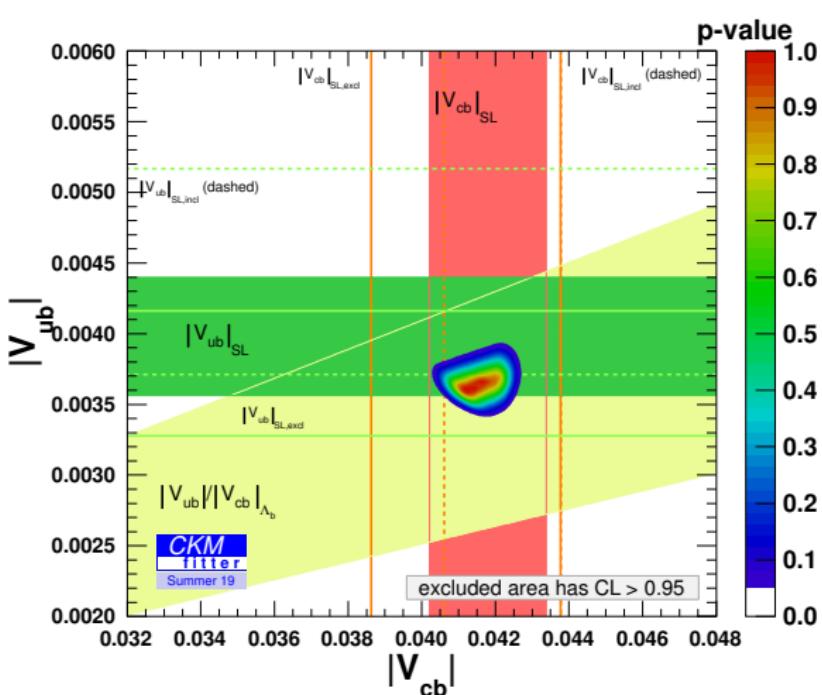
From time to time, question about compatibility of ϵ_K with the rest of the fit, related to the fact that ϵ_K has a strong dependence on

- B_K : role of theoretical uncertainties
- $|V_{cb}|$: inclusive, exclusive or average



- Rfit versus Gaussian treatment of theoretical uncertainties
- agreement of prediction with experiment in both cases

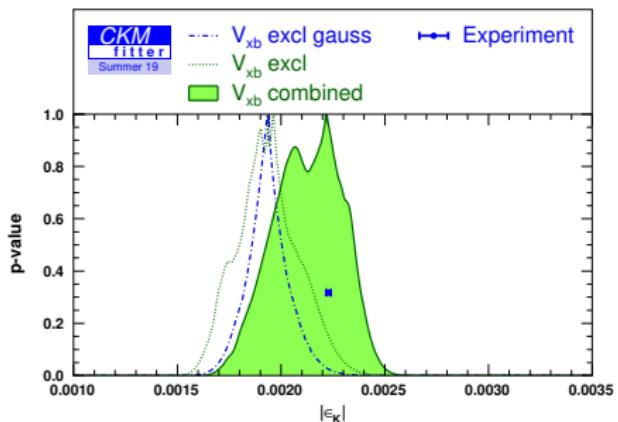
$|V_{cb}|$ and $|V_{ub}|$



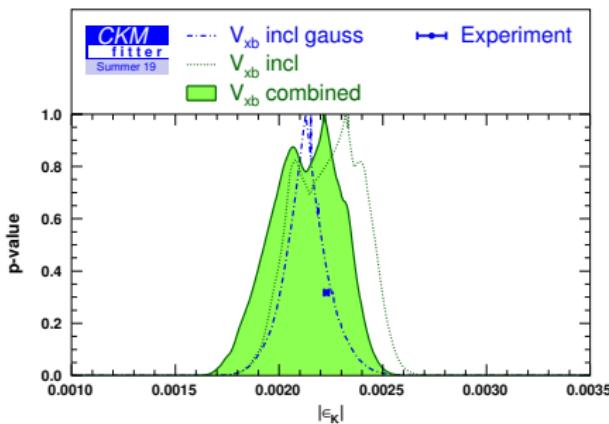
- $|V_{cb}|$ and $|V_{ub}|$ from exclusive and inclusive measurements $b \rightarrow c\ell\nu$ and $b \rightarrow u\ell\nu$ ($\ell = e, \mu$)
- Global fit of CKM favours excl. $|V_{ub}|_{SL}$ but incl. $|V_{cb}|_{SL}$
- $|V_{ub}|$ from $Br(B \rightarrow \tau\nu)$
- $|V_{ub}/V_{cb}|$ from $\Gamma(\Lambda_b \rightarrow p\mu\nu)/\Gamma(\Lambda_b \rightarrow \Lambda_c\mu\nu)$ (large uncertainties)

Ongoing work to understand theoretical uncertainties better

Exclusive versus inclusive for ϵ_K



$|V_{xb}|$ exclusive only



$|V_{xb}|$ inclusive only

- Exclusive slightly off compared to inclusive
- But good agreement in all cases

- $s \rightarrow d\nu\bar{\nu}$ transitions
- relatively clean or very clean probes of the SM

$$\begin{aligned}\mathcal{B}[K^+ \rightarrow \pi^+ \nu\bar{\nu}]_{\text{SM}} &= \kappa_+ (1 + \Delta_{em}) \left[\left(\frac{Im\lambda_t}{\lambda^5} X_t \right)^2 + \left(\frac{Re\lambda_c}{\lambda} (P_c + \delta P_{c,u}) + \frac{Re\lambda_t}{\lambda^5} X_t \right)^2 \right] \\ \mathcal{B}[K_L \rightarrow \pi^0 \nu\bar{\nu}]_{\text{SM}} &= \kappa_L \left(\frac{Im\lambda_t}{\lambda^5} X_t \right)^2,\end{aligned}$$

[Buras et al.; Brod, Gorbahn; Mescia, Smith]

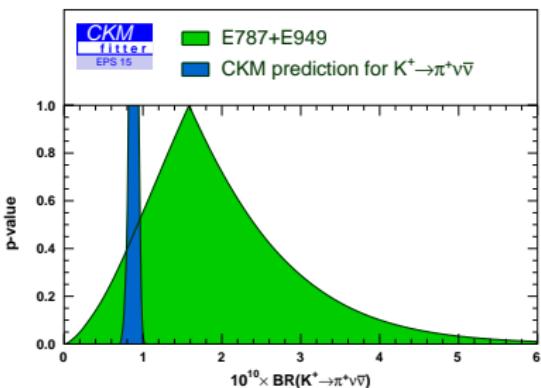
- isospin-breaking parameter $\kappa_{+,L}$ from semileptonic *K* decays
- Δ_{em} electromagnetic correction,
- X_t top-quark contributions, P_c and $\delta P_{c,u}$ light-quark contributions

$$K \rightarrow \pi \nu \bar{\nu}$$

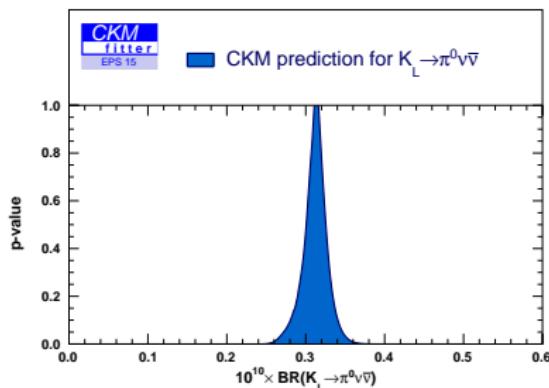
Recent results

- NA62 : bound $\mathcal{B}[K^+ \rightarrow \pi^+ \nu \bar{\nu}] < 2.44 \times 10^{-10}$ at 95% CL
- KOTO : no measurement of $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ yet, but unexpected excess in signal region

Predictions from the global fit



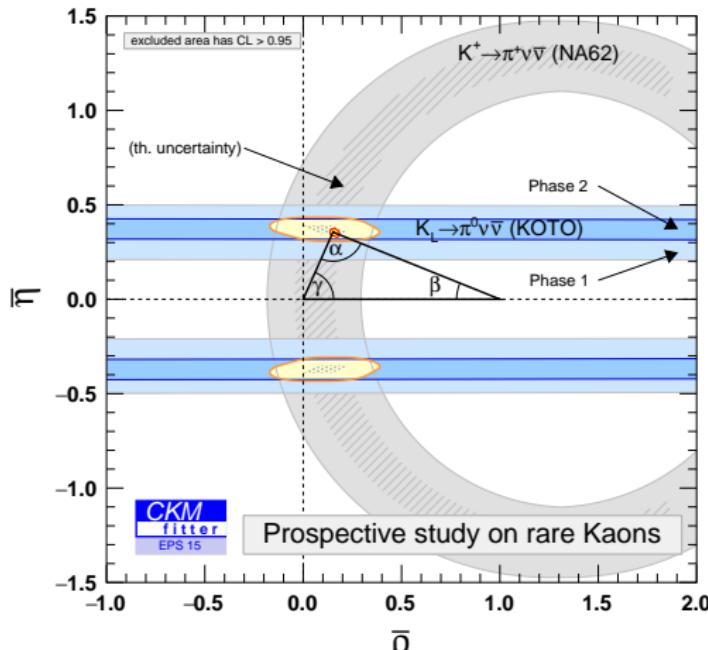
$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (0.89^{+0.09}_{-0.10}) \times 10^{-10}$$



$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (0.31^{+0.02}_{-0.02}) \times 10^{-10}$$

Prospective for rare decays

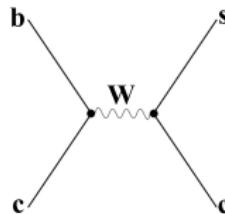
- NA62 : $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ at 10% accuracy
- KOTO : Phase 1 $\sim 3\sigma$ constraint on the branching ratio (SM),
Phase 2 stage with $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ at 10% accuracy



- NA62: in grey the role played by theoretical uncertainties
- KOTO : phases 1 and 2 indicated

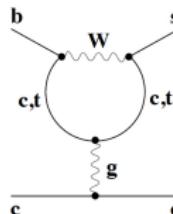
B_s mixing: $B_s \rightarrow J/\psi \phi$

Interference between $B_s \bar{B}_s$ mixing and $\bar{b} \rightarrow \bar{c} s \bar{s}$ decay



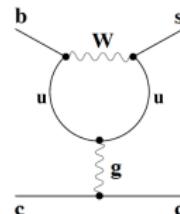
tree

$$V_{cb} V_{cs}^* \\ O(\lambda^2) \text{ real}$$



c,t penguins

$$V_{cb} V_{cs}^* \text{ and } V_{tb} V_{ts}^* \\ O(\alpha_s \lambda^2) \text{ real}$$

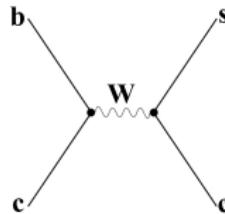


u penguins

$$V_{ub} V_{us}^* \\ O(\alpha_s \lambda^4)$$

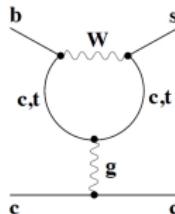
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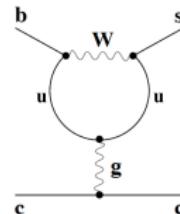
tree

$$V_{cb} V_{cs}^* \\ O(\lambda^2) \text{ real}$$



c,t penguins

$$V_{cb} V_{cs}^* \text{ and } V_{tb} V_{ts}^* \\ O(\alpha_s \lambda^2) \text{ real}$$



u penguins

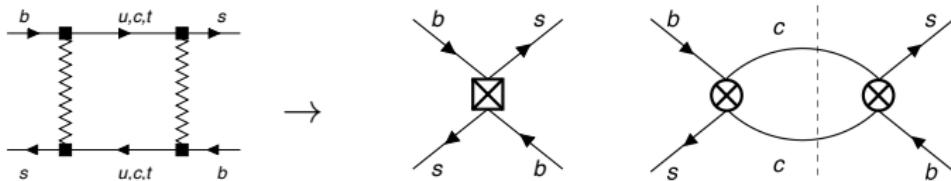
$$V_{ub} V_{us}^* \\ O(\alpha_s \lambda^4)$$

Time-dependent asymmetry yields $S = \sin(\phi_{B_s})$

- $\phi_{B_s} = 2\beta_s = O(\lambda^2)$, and thus very small in SM
- Very strong constraint on NP
- $J/\psi\phi$ not CP-eigenstate
 - 3 possible helicity states (same for J/ψ and ϕ)
 - three amplitudes $A_0, A_\perp, A_{||}$ with definite CP-parity
 - ang. analysis to separate $A_{0,\perp,||}$, determine relative strong phases

$\Delta B = 2$ observables

Eff. Hamiltonian
integrating out
heavy W, Z, t



$$A_{\Delta B=2} = \langle \bar{B} | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B \rangle - \frac{1}{2} \int d^4x d^4y \langle \bar{B} | T \mathcal{H}_{\text{eff}}^{\Delta B=1}(x) \mathcal{H}_{\text{eff}}^{\Delta B=1}(y) | B \rangle$$

- M_{12}^q dominated by **dispersive part of top boxes** [Re[loops]]
 - related to heavy virtual states ($t\bar{t}\dots$)
 - easily affected by NP, e.g., if heavy new particles in the box
- Γ_{12}^q dominated by **absorptive part of charm boxes** [Im[loops]]
 - common B and \bar{B} decay channels into final states with $c\bar{c}$ pair
 - affected by NP if changes in (constrained) tree-level decays

Frequentist approach

$$p = (A, \lambda, \bar{\rho}, \bar{\eta} \dots) = (q, r)$$

- q parameters of interest (CKM), r nuisance parameters (hadronic)
- $\mathcal{O}_{\text{meas}} \pm \sigma_{\mathcal{O}}$ experimental values of observables
- $\mathcal{O}_{\text{th}}(p)$ theoretical description in a given model

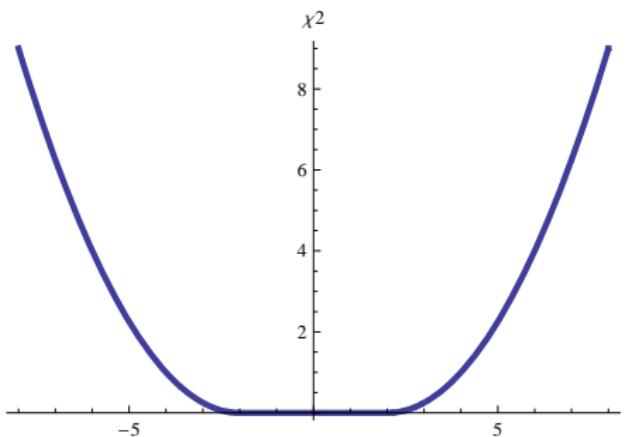
$$\mathcal{L}(p) = \prod_{\mathcal{O}} \mathcal{L}_{\mathcal{O}}(p) \quad T(p) = -2 \ln \mathcal{L}(p) = \sum_{\mathcal{O}} \left(\frac{\mathcal{O}_{\text{th}}(p) - \mathcal{O}_{\text{meas}}}{\sigma_{\mathcal{O}}} \right)^2$$

$$\chi^2(q) = \min_r T(q, r)$$

- Central value: estimator \hat{q} **max likelihood** $\chi^2(\hat{q}) = \min_q \chi^2(q)$
- Range: **confidence level** (p -value) for q_0 computed from $\Delta\chi^2(q_0) = \chi^2(q_0) - \min_q \chi^2(q)$, assuming χ^2 law with $N = \dim(q)$
- Specific (Rfit) treatment of **theoretical uncertainties** modifying \mathcal{L} , and impacting the procedure to average measurements

Theoretical uncertainties

- Observable = CKM \otimes hadronic
- hadronic input often from lattice QCD simulations: $X = X_0 \pm \sigma \pm \Delta$
 - σ statistical, scales with size of sampling, Gaussian model
 - Δ theoretical, dominant for lattice, modelling with no consensus



- CKMfitter: **Rfit approach**
 - modify likelihood $\mathcal{L} = \exp(-\chi^2/2)$
 - χ^2 with flat bottom (theo/syst) and parabolic walls (stat)
 - all values within range of syst treated on same footing
 - averaging procedure designed consistently

- Other approaches: Gaussian (combined in quadrature with statistics), adaptive...

[Charles et al.]

Averaging lattice results

Collecting lattice results

- follow FLAG to exclude limited results
- supplement with more recent published results with error budget

Splitting error estimates into stat and syst

- Stat : essentially related to size of gauge conf
- Syst : fermion action, $a \rightarrow 0$, $L \rightarrow \infty$, mass extrapolations...
added **linearly** using error budget

“Educated Rfit” used to combine the results

- no correlations assumed
- product of (Gaussian + Rfit) likelihoods for central value
- product of Gaussian (stat) likelihoods for stat uncertainty
- syst uncertainty of the combination = most precise method
 - the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
 - best estimate should not be penalized by less precise methods