

# UPDATED RESULTS ON THE CKM MATRIX

Including results presented up to  
CKM 14

*P r e l i m i n a r y*

January 1st, 2015

The CKMfitter Group

## Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach *Rfit*. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

CP VIOLATION AND THE CKM MATRIX:  
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES  
By CKMfitter Group  
Eur. Phys. J. **C41**, 1-131, 2005 [hep-ph/0406184]

## The CKMfitter Group

J. Charles<sup>a</sup>, O. Deschamps<sup>b</sup>, S. Descotes-Genon<sup>c</sup>, H. Lacker<sup>d</sup>, A. Menzel<sup>d</sup>,  
S. Monteil<sup>b</sup>, V. Niess<sup>b</sup>, J. Ocariz<sup>e</sup>, J. Orloff<sup>b</sup>, A. Perez<sup>f</sup>, W. Qian<sup>g</sup>,  
V. Tisserand<sup>g</sup>, K. Trabelsi<sup>h,i</sup>, P. Urquijo<sup>j</sup>, L. Vale Silva<sup>c</sup>

<sup>a</sup>*Aix Marseille Université, Université de Toulon, CNRS, CPT UMR 7332, 13288, Marseille, France*  
*e-mail: charles@cpt.univ-mrs.fr*

<sup>b</sup>*Laboratoire de Physique Corpusculaire de Clermont-Ferrand  
Université Blaise Pascal, 24 Avenue des Landais F-63177 Aubière Cedex, France  
(UMR 6533 du CNRS-IN2P3 associée à l'Université Blaise Pascal)*  
*e-mail: odescham@in2p3.fr, monteil@in2p3.fr, niess@in2p3.fr, orloff@in2p3.fr*

<sup>c</sup>*Laboratoire de Physique Théorique  
Bâtiment 210, Université Paris-Sud 11, F-91405 Orsay Cedex, France  
(UMR 8627 du CNRS associée à l'Université de Paris-Sud 11)*  
*e-mail: Sebastien.Descotes-Genon@th.u-psud.fr, Luiz.Vale@th.u-psud.fr*

<sup>d</sup>*Humboldt-Universität zu Berlin,  
Institut für Physik, Newtonstr. 15, D-12489 Berlin, Germany*  
*e-mail: lacker@physik.hu-berlin.de*

<sup>e</sup>*Laboratoire de Physique Nucléaire et de Hautes Energies,  
IN2P3/CNRS, Université Pierre et Marie Curie Paris 6  
et Université Denis Diderot Paris 7, F-75252 Paris, France*  
*e-mail: Ocariz@in2p3.fr*

<sup>f</sup>*Institut Pluridisciplinaire Hubert Curien, 23 rue du loess - BP28, 67037 Strasbourg cedex 2*  
*e-mail: Luis\_Alejandro.Perez\_Perez@iphc.cnrs.fr*

<sup>g</sup>*Laboratoire d'Annecy-Le-Vieux de Physique des Particules  
9 Chemin de Bellevue, BP 110, F-74941 Annecy-le-Vieux Cedex, France  
(UMR 5814 du CNRS-IN2P3 associée à l'Université de Savoie)*  
*e-mail: qian@lapp.in2p3.fr, tisserav@lapp.in2p3.fr*

<sup>h</sup>*High Energy Accelerator Research Organization, KEK  
1-1 Oho, Tsukuba, Ibaraki 305-0801 Japan*  
<sup>i</sup>*Ecole Polytechnique Fédérale de Lausanne (EPFL), Bâtiment des Sciences Physiques,  
CH-1015 Lausanne, Switzerland*  
*e-mail: karim.trabelsi@kek.jp*

<sup>j</sup>*School of Physics, University of Melbourne, Victoria 3010, Australia*  
*e-mail: Phillip.Urquijo@cern.ch*

## 1 Inputs

Parameter	Value $\pm$ Error(s)	Reference	Errors GS	Errors TH
$ V_{ud} $ (nuclei)	$0.97425 \pm 0 \pm 0.00022$	[1]	-	$\star$
$ V_{us} f_+^{K \rightarrow \pi}(0)$	$0.21664 \pm 0.00048$	[3]	$\star$	$\star$
$ V_{cd} (\nu N)$	$0.230 \pm 0.011$	[3]	$\star$	-
$ V_{cs} (W \rightarrow c\bar{s})$	$0.94^{+0.32}_{-0.26} \pm 0.13$	[3]	$\star$	$\star$
$ V_{ub} $ (semileptonic)	$(3.70 \pm 0.12 \pm 0.26) \times 10^{-3}$	[4–6]	$\star$	$\star$
$ V_{cb} $ (semileptonic)	$(41.00 \pm 0.33 \pm 0.74) \times 10^{-3}$	[4, 6]	$\star$	$\star$
$\mathcal{B}(B^- \rightarrow \tau^-\bar{\nu}_\tau)$	$(1.08 \pm 0.21) \times 10^{-4}$	[4, 7]	$\star$	-
$\mathcal{B}(D_s^- \rightarrow \mu^-\bar{\nu}_\mu)$	$(5.57 \pm 0.24) \times 10^{-3}$	[4]	$\star$	-
$\mathcal{B}(D_s^- \rightarrow \tau^-\bar{\nu}_\tau)$	$(5.55 \pm 0.24) \times 10^{-2}$	[4]	$\star$	-
$\mathcal{B}(D^- \rightarrow \mu^-\bar{\nu}_\mu)$	$(3.74 \pm 0.17) \times 10^{-4}$	[4]	$\star$	$\star$
$\mathcal{B}(K^- \rightarrow e^-\bar{\nu}_e)$	$(1.581 \pm 0.008) \times 10^{-5}$	[3]	$\star$	-
$\mathcal{B}(K^- \rightarrow \mu^-\bar{\nu}_\mu)$	$0.6355 \pm 0.0011$	[3]	$\star$	-
$\mathcal{B}(\tau^- \rightarrow K^-\bar{\nu}_\tau)$	$(0.6955 \pm 0.0096) \times 10^{-2}$	[4]	$\star$	-
$\mathcal{B}(K^- \rightarrow \mu^-\bar{\nu}_\mu)/\mathcal{B}(\pi^- \rightarrow \mu^-\bar{\nu}_\mu)$	$1.3365 \pm 0.0032$	[3]	$\star$	-
$\mathcal{B}(\tau^- \rightarrow K^-\bar{\nu}_\tau)/\mathcal{B}(\tau^- \rightarrow \pi^-\bar{\nu}_\tau)$	$(6.43 \pm 0.09) \times 10^{-2}$	[4]	$\star$	-
$\mathcal{B}(B_s \rightarrow \mu\mu)$	$(2.8^{+0.7}_{-0.6}) \times 10^{-9}$	[8]	$\star$	-
$ V_{cd} f_+^{D \rightarrow \pi}(0)$	$0.148 \pm 0.004$	[9]	$\star$	-
$ V_{cs} f_+^{D \rightarrow K}(0)$	$0.712 \pm 0.007$	[9, 10]	$\star$	-
$ \varepsilon_K $	$(2.228 \pm 0.011) \times 10^{-3}$	[3]	$\star$	-
$\Delta m_d$	$(0.510 \pm 0.003) \text{ ps}^{-1}$	[4]	$\star$	-
$\Delta m_s$	$(17.757 \pm 0.021) \text{ ps}^{-1}$	[4]	$\star$	-
$\sin(2\beta)_{[cc]}$	$0.682 \pm 0.019$	[4]	$\star$	-
$(\phi_s)_{[b \rightarrow c\bar{s}s]}$	$-0.015 \pm 0.035$	[4]	$\star$	-
$S_{\pi\pi}^{+-}, C_{\pi\pi}^{+-}, C_{\pi\pi}^{00}, \mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis	[11–19]	$\star$	-
$S_{\rho\rho,L}^{+-}, C_{\rho\rho,L}^{+-}, S_{\rho\rho}^{00}, C_{\rho\rho}^{00}, \mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[20–26]	$\star$	-
$B^0 \rightarrow (\rho\pi)^0 \rightarrow 3\pi$	Time-dependent Dalitz analysis	[27, 28]	$\star$	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to GLW analysis	[29, 30]	$\star$	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to ADS analysis	[30, 31]	$\star$	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	GGSZ Dalitz analysis	[32]	$\star$	-

Table 1: *Inputs to the standard CKM fit.* If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the “GS” column by a star; measurements or parameters that have systematic theoretical errors are marked in the “TH” column by a star. Upper part: experimental determinations of the CKM matrix elements. Lower part: CP-violation and mixing observables.

Parameter	Value $\pm$ Error(s)	Reference	Errors	
			GS	TH
$\overline{m}_c(m_c)$	$(1.286 \pm 0.013 \pm 0.040) \text{ GeV}$	[33]	*	*
$\overline{m}_t(m_t)$	$(165.95 \pm 0.35 \pm 0.64) \text{ GeV}$	[37]	*	*
$\alpha_s(m_Z)$	$0.1185 \pm 0 \pm 0.0006$	[3]	-	*
<hr/>				
$B_K$	$0.7615 \pm 0.0027 \pm 0.0137$	[2]	*	*
$\kappa_\epsilon$	$0.940 \pm 0.013 \pm 0.023$	[39, 40]	*	*
$\eta_{cc}$	$1.87 \pm 0 \pm 0.76$	[41]	-	*
$\eta_{ct}$	$0.497 \pm 0 \pm 0.047$	[42]	-	*
$\eta_{tt}$	$0.5765 \pm 0 \pm 0.0065$	[43]	-	*
$\eta_B(\overline{\text{MS}})$	$0.5510 \pm 0 \pm 0.0022$	[44, 45]	-	*
$f_{B_s}$	$(225.6 \pm 1.1 \pm 5.4) \text{ MeV}$	[2]	*	*
$B_s$	$1.320 \pm 0.017 \pm 0.030$	[2]	*	*
$f_{B_s}/f_{B_d}$	$1.205 \pm 0.004 \pm 0.007$	[2]	*	*
$B_s/B_d$	$1.023 \pm 0.013 \pm 0.014$	[2]	*	*
<hr/>				
$f_K$	$(155.2 \pm 0.2 \pm 0.6) \text{ MeV}$	[2]	*	*
$f_K/f_\pi$	$1.1942 \pm 0.0009 \pm 0.0030$	[2]	*	*
$f_{D_s}$	$(245.3 \pm 0.5 \pm 4.5) \text{ MeV}$	[2]	*	*
$f_{D_s}/f_D$	$1.201 \pm 0.004 \pm 0.010$	[2]	*	*
<hr/>				
$f_+^{K \rightarrow \pi}(0)$	$0.9641 \pm 0.0015 \pm 0.0045$	[2]	*	*
$f_+^{D \rightarrow \pi}(0)$	$0.666 \pm 0.020 \pm 0.048$	[2]	*	*
$f_+^{D \rightarrow K}(0)$	$0.747 \pm 0.011 \pm 0.034$	[2]	*	*

Table 2: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the “GS” column by a star; measurements or parameters that have systematic theoretical errors are marked in the “TH” column by a star. Upper part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

## 2 Lattice QCD averages

Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

### 2.1 Method of averaging

We collect the relevant calculations of the quantity that we are interested in: we take only unquenched results with 2 or 2+1 dynamical fermions, even those from proceedings without a companion article. For the calculations published before the end of November 2013, we have followed the classification of the Flavour Lattice Averaging Group [53] and kept only results with green squares. However, we stress that we perform our averages in a different manner from FLAG.

In these results, we separate the error estimates into a Gaussian part and a flat part that is treated à la Rfit. The Gaussian part collects the uncertainties from purely statistical origin, but also the systematics that can be controlled and treated in a similar way (e.g., interpolation or fitting in some cases). The remaining systematics constitute the Rfit error. If there are several sources of error in the Rfit category, we add them linearly<sup>1</sup>.

The Rfit model is simple but also very strict. It amounts to assuming that the theoretical uncertainty is rigorously constrained by a mathematical bound that is our only piece of information. If Rfit is taken stricto sensu and the individual likelihoods are combined in the usual way (by multiplication), the final uncertainty can be underestimated, in particular in the case of marginally compatible values.

We correct this effect by adopting the following averaging recipe. The central value is obtained by combining the whole likelihoods. Then we combine the Gaussian uncertainties by combining likelihoods restricted to their Gaussian part. Finally we assign to this combination the smallest of the individual Rfit uncertainties. The underlying idea is twofold:

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
- this best estimate should not be penalized by less precise methods (as it would happen be the case if one would take the dispersion of the individual central values as a guess of the combined theoretical uncertainty).

It should be stressed that the concept of a theoretical uncertainty is ill-defined, and the combination of them even more. Thus our approach is only one among the alternatives that can be found in the literature. In contrast to some of the latter, ours is algorithmic and can be reproduced.

## 3 Decay constants

### 3.1 Light mesons

$$f_K$$

---

<sup>1</sup>keeping in mind that in many papers in the literature, this combination is done in quadrature and the splitting between different sources is not published.

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC09	[60]	2	158.1	0.8	3.1
HPQCD07	[61]	2+1	157	0.6	3.3
MILC10	[62]	2+1	156.1	0.4	$^{+0.6}_{-0.9}$
LVdW11	[63]	2+1	156.8	1.0	3.3
RBC-UKQCD12	[64]	2+1	152.4	3.0	2.2
HPQCD13	[65]	2+1+1	155.4	0.2	0.6
ETMC13	[66]	2+1+1	155.6	1.6	2.2
Our average			155.2	0.2	0.6

$f_K/f_\pi$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC09	[60]	2	1.210	0.006	0.024
HPQCD/UKQCD07	[61]	2+1	1.189	0.002	0.014
MILC10	[62]	2+1	1.197	0.002	$^{+0.003}_{-0.007}$
BMW10	[67]	2+1	1.192	0.007	0.013
LVdW11	[63]	2+1	1.202	0.011	0.024
RBC-UKQCD12	[64]	2+1	1.1991	0.0116	0.0185
HPQCD13	[65]	2+1+1	1.1938	0.0015	0.0032
MILC13	[68]	2+1+1	1.1969	0.0026	0.0052
ETMC13	[66]	2+1+1	1.193	0.013	0.019
Our average			1.1942	0.0009	0.0030

Results have been corrected to express results in terms of the decay constants defined in QCD (electromagnetic corrections are applied at the level of the branching ratios).

### 3.2 Charmed mesons

$f_{D_s}$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC09	[60]	2	244	3	9
HPQCD10	[69]	2+1	248.0	1.4	4.5
FNAL-MILC11	[70]	2+1	260.1	8.9	16.2
FNAL-MILC12	[71]	2+1+1	246.4	0.5	5.6
ETMC13	[66]	2+1+1	242.1	7.6	4.6
Our average			245.3	0.5	4.5

$f_{D_s}/f_D$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC09	[60]	2	1.24	0.03	0.01
FNAL-MILC11	[70]	2+1	1.188	0.014	0.054
HPQCD12	[72]	2+1	1.187	0.004	0.023
FNAL-MILC12	[71]	2+1+1	1.175	0.016	0.018
ETMC13	[66]	2+1+1	1.199	0.017	0.023
Our average			1.201	0.004	0.010

### 3.3 Beauty mesons

$f_{B_s}$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC13	[73]	2	228	5	9
HPQCD11	[75]	2+1	225.0	2.9	5.4
FNAL-MILC11	[70]	2+1	242.0	5.1	21.2
HPQCD12	[76]	2+1	228.0	1.4	17.5
HPQCD13	[77]	2+1+1	224.0	2.5	7.2
Our average			225.6	1.1	5.4

$f_{B_s}/f_B$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC13	[73]	2	1.206	0.010	0.026
FNAL-MILC11	[70]	2+1	1.229	0.013	0.046
HPQCD12	[76]	2+1	1.188	0.012	0.025
HPQCD13	[77]	2+1	1.205	0.004	0.007
2 1.203 0.062 0.019 Our average			1.205	0.004	0.007

## 4 Semileptonic form factors

### 4.1 $K \rightarrow \pi \ell \nu$

$f_+(0)$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC09	[78]	2	0.9560	0.0057	0.0118
MILC12	[79]	2+1	0.9667	0.0023	0.0077
RBC-UKQCD13	[80]	2+1	0.9670	0.0020	$^{+0.024}_{-0.066}$
Our average			0.9641	0.0015	0.0045

Combining with  $|V_{us}|f_+(0) = 0.2166(5)$  from ref. [81], we get  $|V_{us}| = 0.2247 \pm 0.0006 \pm 0.0011$ .

### 4.2 $D \rightarrow \pi \ell \nu$

$f_+(0)$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC11	[82]	2	0.65	0.06	0.12
HPQCD11	[83]	2+1	0.666	0.021	0.048
Our average			0.666	0.020	0.048

### 4.3 $D \rightarrow K\ell\nu$

$f_+(0)$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC11	[82]	2	0.76	0.05	0.11
HPQCD10	[84]	2+1	0.747	0.011	0.034
Our average			0.747	0.011	0.034

## 5 Meson mixing

### 5.1 Kaon mixing

$B_K^{\overline{\text{MS}}}(2\text{GeV})$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC 10	[85]	2	0.532	0.019	0.026
LVdW11	[63]	2+1	0.5572	0.0028	0.0257
BMW11	[86]	2+1	0.5644	0.0059	0.0100
RBC-UKQCD12	[64]	2+1	0.549	0.010	0.030
SWME14	[87]	2+1	0.5388	0.0034	0.0442
Our average for $B_K^{\overline{\text{MS}}}(2\text{GeV})$			0.5562	0.0020	0.0100
Our average for $\hat{B}_K$			0.7615	0.0027	0.0137

### 5.2 $B_{d,s}$ mixing

$\hat{B}_{B_s}$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC13	[73]	2	1.32	0.04	0.03
HPQCD09	[88]	2+1	1.326	0.018	0.040
Our average			1.32	0.017	0.030

Ref. [88] provide only  $f_{B_s}$  and  $f_{B_s}\sqrt{\hat{B}_{B_s}}$ , and we assumed that the systematics were completely correlated to extract  $\hat{B}_{B_s}$ .

$\hat{B}_{B_s}/\hat{B}_{B_d}$

Reference	Article	$N_f$	Mean	Stat	Syst
ETMC13	[73]	2	1.007	0.015	0.014
HPQCD09	[88]	2+1	1.053	0.025	0.023
FNAL/MILC12	[89]	2+1	1.064	0.076	0.193
Our average			1.023	0.013	0.014

Refs. [88] and [89] provide only  $\xi$  and  $f_{B_s}/f_{B_d}$ . For Refs. [88], we have extracted  $\hat{B}_{B_s}/\hat{B}_{B_d}$  in both cases assuming a total correlation in the systematics of  $\xi$  and  $\hat{B}_{B_s}/\hat{B}_{B_d}$ . For Ref. [89], we have considered all uncertainties as uncorrelated, as the studies of the decay constants and the bag parameters have been performed in different settings, with different categories of systematics.

## 6 Results

Observable	central $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
$A$	$0.810^{+0.018}_{-0.024}$	$0.810^{+0.025}_{-0.030}$	$0.810^{+0.032}_{-0.035}$
$\lambda$	$0.22548^{+0.00068}_{-0.00034}$	$0.22548^{+0.00096}_{-0.00068}$	$0.2255^{+0.0010}_{-0.0010}$
$\bar{\rho}$	$0.145^{+0.013}_{-0.007}$	$0.145^{+0.032}_{-0.015}$	$0.145^{+0.042}_{-0.022}$
$\bar{\eta}$	$0.343^{+0.011}_{-0.012}$	$0.343^{+0.022}_{-0.025}$	$0.343^{+0.034}_{-0.036}$
$J [10^{-5}]$	$2.96^{+0.19}_{-0.17}$	$2.96^{+0.31}_{-0.22}$	$2.96^{+0.42}_{-0.27}$
$\sin 2\alpha$	$-0.036^{+0.042}_{-0.082}$	$-0.036^{+0.084}_{-0.189}$	$-0.04^{+0.12}_{-0.25}$
$\sin 2\alpha$ (!)	$-0.053^{+0.046}_{-0.146}$	$-0.053^{+0.090}_{-0.209}$	$-0.05^{+0.13}_{-0.26}$
$\sin 2\beta$	$0.692^{+0.018}_{-0.019}$	$0.692^{+0.037}_{-0.037}$	$0.692^{+0.055}_{-0.052}$
$\sin 2\beta$ (!)	$0.771^{+0.017}_{-0.041}$	$0.771^{+0.034}_{-0.103}$	$0.771^{+0.050}_{-0.141}$
$\alpha [\circ]$	$91.0^{+2.3}_{-1.2}$	$91.0^{+5.5}_{-2.4}$	$91.0^{+7.2}_{-3.6}$
$\alpha [\circ]$ (!)	$91.5^{+4.2}_{-1.3}$	$91.5^{+6.1}_{-2.6}$	$91.5^{+7.6}_{-3.8}$
$\alpha [\circ]$ (dir. meas.)	$87.7^{+3.5}_{-3.3}    -1.1^{+3.8}_{-4.0}$	$87.7^{+10.1}_{-6.5}    -1.1^{+7.8}_{-9.0}$	$87.7^{+16.1}_{-9.8}    -1^{+12}_{-15}$
$\beta [\circ]$	$21.89^{+0.74}_{-0.77}$	$21.9^{+1.5}_{-1.4}$	$21.9^{+2.3}_{-2.0}$
$\beta [\circ]$ (!)	$25.22^{+0.78}_{-1.79}$	$25.2^{+1.6}_{-4.3}$	$25.2^{+2.4}_{-5.7}$
$\beta [\circ]$ (dir. meas.)	$21.50^{+0.75}_{-0.74}$	$21.5^{+1.5}_{-1.5}$	$21.5^{+2.3}_{-2.2}$
$\gamma [\circ]$	$67.08^{+0.97}_{-2.17}$	$67.1^{+2.0}_{-5.0}$	$67.1^{+3.0}_{-6.4}$
$\gamma [\circ]$ (!)	$66.9^{+1.0}_{-3.7}$	$66.9^{+2.0}_{-5.5}$	$66.9^{+3.1}_{-6.6}$
$\gamma [\circ]$ (dir. meas.)	$73.2^{+6.3}_{-7.0}$	$73^{+13}_{-15}$	$73^{+20}_{-24}$
$(\sin 2\beta + \gamma)$	$0.934^{+0.015}_{-0.011}$	$0.934^{+0.033}_{-0.023}$	$0.934^{+0.043}_{-0.036}$
$R_u$	$0.373^{+0.012}_{-0.012}$	$0.373^{+0.024}_{-0.023}$	$0.373^{+0.036}_{-0.032}$
$R_t$	$0.9212^{+0.0063}_{-0.0141}$	$0.921^{+0.013}_{-0.033}$	$0.921^{+0.020}_{-0.041}$
$\beta_s \equiv -\arg(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*})$ [rad]	$0.01826^{+0.00059}_{-0.00064}$	$0.0183^{+0.0012}_{-0.0013}$	$0.0183^{+0.0018}_{-0.0019}$
$\sin 2\beta_s$	$0.0365^{+0.0012}_{-0.0013}$	$0.0365^{+0.0024}_{-0.0026}$	$0.0365^{+0.0036}_{-0.0038}$

(!) means that the measurement was not included in the fit, || indicates the union of the confidence intervals considered.

<b>Observable</b>	<b>central</b> $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
$ V_{ud} $	$0.974242^{+0.000079}_{-0.000158}$	$0.97424^{+0.00016}_{-0.00022}$	$0.97424^{+0.00023}_{-0.00024}$
$ V_{us} $	$0.22548^{+0.00068}_{-0.00034}$	$0.22548^{+0.00096}_{-0.00068}$	$0.2255^{+0.0010}_{-0.0010}$
$ V_{ub} $	$0.00355^{+0.00017}_{-0.00015}$	$0.00355^{+0.00029}_{-0.00022}$	$0.00355^{+0.00041}_{-0.00029}$
$ V_{cd} $	$0.22534^{+0.00068}_{-0.00034}$	$0.22534^{+0.00096}_{-0.00068}$	$0.2253^{+0.0011}_{-0.0010}$
$ V_{cs} $	$0.97341^{+0.00011}_{-0.00018}$	$0.97341^{+0.00019}_{-0.00025}$	$0.97341^{+0.00027}_{-0.00028}$
$ V_{cb} $	$0.04117^{+0.00090}_{-0.00114}$	$0.0412^{+0.0012}_{-0.0014}$	$0.0412^{+0.0016}_{-0.0016}$
$ V_{td} $	$0.00855^{+0.00021}_{-0.00027}$	$0.00855^{+0.00030}_{-0.00041}$	$0.00855^{+0.00039}_{-0.00051}$
$ V_{ts} $	$0.04043^{+0.00088}_{-0.00112}$	$0.0404^{+0.0012}_{-0.0014}$	$0.0404^{+0.0015}_{-0.0016}$
$ V_{tb} $	$0.999146^{+0.000046}_{-0.000038}$	$0.999146^{+0.000056}_{-0.000052}$	$0.999146^{+0.000065}_{-0.000066}$
$ V_{ud} $ (!)	$0.974243^{+0.000079}_{-0.000156}$	$0.97424^{+0.00016}_{-0.00023}$	$0.97424^{+0.00024}_{-0.00030}$
$ V_{us} $ (!)	$0.224489^{+0.001589}_{-0.000067}$	$0.22449^{+0.00201}_{-0.00013}$	$0.22449^{+0.00208}_{-0.00020}$
$ V_{ub} $ (!)	$0.003455^{+0.000227}_{-0.000095}$	$0.00346^{+0.00035}_{-0.00018}$	$0.00346^{+0.00048}_{-0.00026}$
$ V_{cb} $ (!)	$0.0412^{+0.0026}_{-0.0013}$	$0.0412^{+0.0029}_{-0.0016}$	$0.0412^{+0.0033}_{-0.0018}$
$\Delta m_d$ [ps $^{-1}$ ] (!)	$0.566^{+0.035}_{-0.043}$	$0.566^{+0.062}_{-0.078}$	$0.566^{+0.090}_{-0.116}$
$\Delta m_s$ [ps $^{-1}$ ] (!)	$16.3^{+1.1}_{-1.1}$	$16.3^{+2.4}_{-1.5}$	$16.3^{+3.5}_{-1.9}$
$ \epsilon_K $ [10 $^{-3}$ ] (!)	$2.03^{+0.59}_{-0.55}$	$2.03^{+0.71}_{-0.64}$	$2.03^{+0.83}_{-0.70}$
$m_t$ [GeV/c $^2$ ] (!)	$157.4^{+7.7}_{-2.2}$	$157.4^{+22.1}_{-4.4}$	$157.4^{+29.4}_{-6.6}$
$B_K$ (!)	$0.86^{+0.27}_{-0.20}$	$0.86^{+0.34}_{-0.23}$	$0.86^{+0.39}_{-0.26}$
$f_{B_s}/f_{B_d}$ (!)	$1.246^{+0.042}_{-0.031}$	$1.246^{+0.073}_{-0.078}$	$1.25^{+0.10}_{-0.12}$
$f_{B_s}$ (!)	$0.2360^{+0.0031}_{-0.0164}$	$0.2360^{+0.0053}_{-0.0189}$	$0.2360^{+0.0076}_{-0.0211}$
$B_{B_s}/B_{B_d}$ (!)	$1.135^{+0.051}_{-0.070}$	$1.14^{+0.10}_{-0.14}$	$1.14^{+0.16}_{-0.22}$
$B_{B_s}$ (!)	$1.295^{+0.211}_{-0.073}$	$1.295^{+0.243}_{-0.097}$	$1.30^{+0.27}_{-0.12}$

(!) means that the measurement was not included in the fit.

<b>Observable</b>	<b>central</b>	$\pm \text{ CL} \equiv 1\sigma$	$\pm \text{ CL} \equiv 2\sigma$	$\pm \text{ CL} \equiv 3\sigma$
$\mathcal{B}(B^+ \rightarrow \tau\nu) [10^{-4}]$	$0.810^{+0.059}_{-0.086}$	$0.81^{+0.12}_{-0.13}$	$0.81^{+0.19}_{-0.17}$	
$\mathcal{B}(B^+ \rightarrow \tau\nu) [10^{-4}] (!)$	$0.758^{+0.080}_{-0.059}$	$0.76^{+0.15}_{-0.10}$	$0.76^{+0.22}_{-0.13}$	
$\mathcal{B}(B^+ \rightarrow \mu\nu) [10^{-6}]$	$0.364^{+0.027}_{-0.038}$	$0.364^{+0.055}_{-0.060}$	$0.364^{+0.085}_{-0.077}$	
$\mathcal{B}(B^+ \rightarrow e\nu) [10^{-11}]$	$0.851^{+0.063}_{-0.090}$	$0.85^{+0.13}_{-0.14}$	$0.85^{+0.20}_{-0.18}$	
$\mathcal{B}(B_d \rightarrow e^+e^-) [10^{-15}]$	$2.29^{+0.14}_{-0.16}$	$2.29^{+0.20}_{-0.23}$	$2.29^{+0.25}_{-0.27}$	
$\mathcal{B}(B_d \rightarrow \mu^+\mu^-) [10^{-11}]$	$9.80^{+0.62}_{-0.68}$	$9.80^{+0.86}_{-0.98}$	$9.8^{+1.1}_{-1.2}$	
$\mathcal{B}(B_s \rightarrow e^+e^-) [10^{-14}]$	$7.64^{+0.46}_{-0.43}$	$7.64^{+0.59}_{-0.52}$	$7.64^{+0.71}_{-0.61}$	
$\mathcal{B}(B_s \rightarrow \mu^+\mu^-) [10^{-9}]$	$3.26^{+0.20}_{-0.18}$	$3.26^{+0.25}_{-0.22}$	$3.26^{+0.31}_{-0.26}$	
$\mathcal{B}(B_s \rightarrow \mu^+\mu^-) [10^{-9}] (!)$	$3.34^{+0.13}_{-0.25}$	$3.34^{+0.18}_{-0.30}$	$3.34^{+0.23}_{-0.34}$	
$\mathcal{B}(D_s^+ \rightarrow \tau^+\nu) [10^{-2}] (!)$	$5.187^{+0.021}_{-0.115}$	$5.187^{+0.042}_{-0.296}$	$5.187^{+0.063}_{-0.400}$	
$\mathcal{B}(D_s^+ \rightarrow \mu^+\nu) [10^{-3}] (!)$	$5.314^{+0.021}_{-0.093}$	$5.314^{+0.043}_{-0.260}$	$5.314^{+0.064}_{-0.403}$	
$\mathcal{B}(D^+ \rightarrow \mu^+\nu) [10^{-4}] (!)$	$3.91^{+0.11}_{-0.11}$	$3.91^{+0.14}_{-0.19}$	$3.91^{+0.18}_{-0.29}$	
$\mathcal{B}(K^+ \rightarrow \mu^+\nu) [10^{-1}] (!)$	$6.364^{+0.027}_{-0.030}$	$6.364^{+0.048}_{-0.060}$	$6.364^{+0.067}_{-0.089}$	
$\mathcal{B}(K^+ \rightarrow e^+\nu) [10^{-5}] (!)$	$1.5690^{+0.0047}_{-0.0047}$	$1.5690^{+0.0093}_{-0.0093}$	$1.569^{+0.013}_{-0.013}$	
$\mathcal{B}(\tau^+ \rightarrow K^+\nu) [10^{-3}] (!)$	$7.178^{+0.015}_{-0.015}$	$7.178^{+0.031}_{-0.031}$	$7.178^{+0.046}_{-0.046}$	
$\Delta\Gamma_d (\text{ps}^{-1}) (!)$	$0.0040^{+0.0017}_{-0.0024}$	$0.0040^{+0.0027}_{-0.0030}$	$0.0040^{+0.0036}_{-0.0040}$	
$\Delta\Gamma_s (\text{ps}^{-1}) (!)$	$0.120^{+0.043}_{-0.045}$	$0.120^{+0.048}_{-0.051}$	$0.120^{+0.054}_{-0.057}$	
$\Delta\Gamma_s (\text{ps}^{-1})$	$0.081^{+0.006}_{-0.005}$	$0.0813^{+0.0118}_{-0.0093}$	$0.081^{+0.018}_{-0.014}$	
$a_{\text{SL}}^d [10^{-4}] (!)$	$-6.5^{+1.8}_{-1.9}$	$-6.5^{+2.2}_{-2.4}$	$-6.5^{+2.5}_{-3.0}$	
$a_{\text{SL}}^d [10^{-4}]$	$-5.1^{+0.4}_{-3.3}$	$-5.1^{+0.7}_{-3.8}$	$-5.1^{+1.1}_{-4.4}$	
$a_{\text{SL}}^s [10^{-4}] (!)$	$0.29^{+0.08}_{-0.08}$	$0.29^{+0.11}_{-0.10}$	$0.29^{+0.13}_{-0.11}$	
$a_{\text{SL}}^s [10^{-4}]$	$0.23^{+0.15}_{-0.02}$	$0.23^{+0.17}_{-0.03}$	$0.23^{+0.20}_{-0.05}$	
$A_{\text{SL}} [10^{-4}] (!)$	$-3.4^{+1.0}_{-1.1}$	$-3.4^{+1.2}_{-1.4}$	$-3.4^{+1.4}_{-1.8}$	
$A_{\text{SL}} [10^{-4}]$	$-4.2^{+1.7}_{-0.3}$	$-4.2^{+2.0}_{-0.7}$	$-4.2^{+2.1}_{-1.1}$	
$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) [10^{-10}] (!)$	$0.85^{+0.13}_{-0.12}$	$0.85^{+0.17}_{-0.15}$	$0.85^{+0.20}_{-0.19}$	
$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) [10^{-10}]$	$0.93^{+0.05}_{-0.19}$	$0.93^{+0.08}_{-0.24}$	$0.93^{+0.12}_{-0.27}$	
$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu}) [10^{-10}] (!)$	$0.29^{+0.04}_{-0.05}$	$0.29^{+0.07}_{-0.09}$	$0.29^{+0.10}_{-0.11}$	

(!) means that the measurement was not included in the fit.

## 7 New physics in $\Delta F = 2$

We follow refs. [40, 46], and consider Scenario I where New Physics enters  $B_d$  and  $B_s$  mixings through  $M_{12}^q = M_{12;SM}^q \Delta_q$ . The two NP complex parameters  $\Delta_q$  are uncorrelated (Scenario I). In addition to the inputs presented above and below, we use the additional theoretical inputs discussed in refs. [40, 46] necessary to describe NP in  $\Delta F = 2$ .

### 7.1 Inputs

Parameter	Value $\pm$ Error(s)	Reference	Errors	
			GS	TH
$A_{SL}$	$(-47 \pm 17) \times 10^{-4}$	[54]		
$a_{SL}^s$	$(1 \pm 20) \cdot 10^{-4}$	[4, 47, 57]	*	-
$a_{SL}^d$	$(-48 \pm 48) \cdot 10^{-4}$	[58, 59]	*	-
$\Delta\Gamma_s$ [ps $^{-1}$ ]	$0.081 \pm 0.006$	[4]	*	-
$\tilde{B}_{S,B_s}/\tilde{B}_{S,B_d}$	$1.01 \pm 0.02 \pm 0.02$	[73]	*	*
$\tilde{B}_{S,B_s}(m_b)$	$0.89 \pm 0.10 \pm 0.09$	[73]	*	*

The DØ result for  $A_{SL}$  in ref. [55] is corrected to take into SM contributions neglected beforehand (standard model CP violation in interference of decays with and without mixing of  $B_0$  to flavor non-specific states), based on ref. [55]. This estimate was later criticised in ref. [56] as being too large a correction. In order to take into account an underestimation of the systematics attached to this estimate, we consider the results both with and without including  $A_{SL}$  in the fit.

## 7.2 Results without $A_{SL}$

Observable	central	$\pm \text{ CL} \equiv 1\sigma$	$\pm \text{ CL} \equiv 2\sigma$	$\pm \text{ CL} \equiv 3\sigma$
$\text{Re}(\Delta_d)$		$0.94^{+0.18}_{-0.15}$	$0.94^{+0.27}_{-0.20}$	$0.94^{+0.39}_{-0.23}$
$\text{Im}(\Delta_d)$		$-0.114^{+0.115}_{-0.052}$	$-0.114^{+0.181}_{-0.099}$	$-0.11^{+0.22}_{-0.15}$
$ \Delta_d $		$0.95^{+0.18}_{-0.15}$	$0.95^{+0.27}_{-0.20}$	$0.95^{+0.39}_{-0.24}$
$\phi_d^\Delta$ [deg]		$-6.9^{+6.9}_{-2.2}$	$-6.9^{+11.1}_{-4.3}$	$-6.9^{+13.6}_{-6.5}$
$\text{Re}(\Delta_s)$		$1.05^{+0.14}_{-0.13}$	$1.05^{+0.17}_{-0.16}$	$1.05^{+0.20}_{-0.18}$
$\text{Im}(\Delta_s)$		$0.028^{+0.039}_{-0.037}$	$0.028^{+0.082}_{-0.075}$	$0.03^{+0.12}_{-0.12}$
$ \Delta_s $		$1.05^{+0.14}_{-0.13}$	$1.05^{+0.17}_{-0.16}$	$1.05^{+0.20}_{-0.18}$
$\phi_s^\Delta$ [deg]		$1.5^{+2.3}_{-2.4}$	$1.5^{+4.7}_{-4.8}$	$1.5^{+6.9}_{-7.1}$
$\phi_d^\Delta + 2\beta$ [deg] (!)		$46.^{+13.}_{-12.}$	$46.^{+27.}_{-24.}$	$46.^{+44.}_{-39.}$
$\phi_s^\Delta - 2\beta_s$ [deg] (!)		$-49.^{+43.}_{-16.}$	$-49.^{+93.}_{-19.}$	$-49.^{+111.}_{-22.}$
$A_{SL}$ [ $10^{-4}$ ] (!)		$-7.1^{+3.7}_{-4.3}$	$-7.1^{+7.7}_{-6.9}$	$-7.1^{+10.2}_{-9.3}$
$A_{SL}$ [ $10^{-4}$ ]		$-7.1^{+3.7}_{-4.3}$	$-7.1^{+7.7}_{-6.9}$	$-7.1^{+10.2}_{-9.3}$
$a_{SL}^d$ [ $10^{-4}$ ] (!)		$121^{+35}_{-43}$ or $-17.3^{+7.6}_{-5.9}$	$121^{+47}_{-50}$ or $-17.3^{+16.0}_{-10.0}$	$121^{+58}_{-58}$ or $-17^{+21}_{-14}$
$a_{SL}^s$ [ $10^{-4}$ ] (!)		$1.6^{+1.9}_{-1.9}$	$1.6^{+3.8}_{-3.8}$	$1.6^{+5.8}_{-5.7}$
$\Delta\Gamma_d$ [ps $^{-1}$ ]		$0.00284^{+0.00184}_{-0.00062}$	$0.00284^{+0.00226}_{-0.00087}$	$0.0028^{+0.0027}_{-0.0011}$
$\Delta\Gamma_s$ [ps $^{-1}$ ] (!)		$0.090^{+0.082}_{-0.024}$	$0.090^{+0.090}_{-0.030}$	$0.090^{+0.097}_{-0.036}$
$\Delta\Gamma_s$ [ps $^{-1}$ ]		$0.0813^{+0.0063}_{-0.0063}$	$0.081^{+0.013}_{-0.012}$	$0.081^{+0.019}_{-0.018}$
$B \rightarrow \tau\nu$ [ $10^{-4}$ ] (!)		$0.688^{+0.380}_{-0.048}$	$0.688^{+0.459}_{-0.095}$	$0.69^{+0.53}_{-0.14}$
$B \rightarrow \tau\nu$ [ $10^{-4}$ ]		$1.029^{+0.062}_{-0.201}$	$1.03^{+0.13}_{-0.38}$	$1.03^{+0.19}_{-0.44}$

(!) means that the measurement was not included in the fit, || indicates the union of the confidence intervals considered.

### 7.3 Results with $A_{SL}$

Observable	central $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
$\text{Re}(\Delta_d)$	$0.883^{+0.217}_{-0.093}$	$0.88^{+0.31}_{-0.14}$	$0.88^{+0.43}_{-0.18}$
$\text{Im}(\Delta_d)$	$-0.113^{+0.075}_{-0.052}$	$-0.11^{+0.15}_{-0.10}$	$-0.11^{+0.20}_{-0.15}$
$ \Delta_d $	$0.890^{+0.220}_{-0.096}$	$0.89^{+0.31}_{-0.15}$	$0.89^{+0.43}_{-0.19}$
$\phi_d^\Delta$ [deg]	$-7.3^{+4.7}_{-2.1}$	$-7.3^{+9.8}_{-4.2}$	$-7.3^{+12.9}_{-6.3}$
$\text{Re}(\Delta_s)$	$1.008^{+0.173}_{-0.093}$	$1.01^{+0.21}_{-0.12}$	$1.01^{+0.24}_{-0.14}$
$\text{Im}(\Delta_s)$	$0.023^{+0.038}_{-0.035}$	$0.023^{+0.080}_{-0.073}$	$0.02^{+0.12}_{-0.11}$
$ \Delta_s $	$1.010^{+0.171}_{-0.095}$	$1.01^{+0.20}_{-0.12}$	$1.01^{+0.23}_{-0.14}$
$\phi_s^\Delta$ [deg]	$1.3^{+2.3}_{-2.3}$	$1.3^{+4.7}_{-4.7}$	$1.3^{+6.9}_{-7.0}$
$\phi_d^\Delta + 2\beta$ [deg] (!)	$37.9^{+9.7}_{-13.1}$	$38.^{+21.}_{-25.}$	$38.^{+35.}_{-40.}$
$\phi_s^\Delta - 2\beta_s$ [deg] (!)	$-61.2^{+13.1}_{-4.8}$	$-61.2^{+39.8}_{-7.6}$	$-61.^{+77.}_{-10.}$
$A_{SL}$ [ $10^{-4}$ ] (!)	$-7.1^{+3.7}_{-4.3}$	$-7.1^{+7.7}_{-6.9}$	$-7.1^{+10.2}_{-9.3}$
$A_{SL}$ [ $10^{-4}$ ]	$-10.4^{+4.7}_{-2.2}$	$-10.4^{+8.7}_{-4.5}$	$-10.4^{+12.1}_{-6.9}$
$a_{SL}^d$ [ $10^{-4}$ ] (!)	$-20.7^{+6.8}_{-3.8}$	$-20.7^{+14.4}_{-7.7}$	$-21^{+22}_{-12}$
$a_{SL}^s$ [ $10^{-4}$ ] (!)	$1.5^{+1.9}_{-1.9}$	$1.5^{+3.8}_{-3.8}$	$1.5^{+5.7}_{-5.7}$
$\Delta\Gamma_d$ [ps $^{-1}$ ]	$0.00417^{+0.00053}_{-0.00185}$	$0.00417^{+0.00095}_{-0.00217}$	$0.0042^{+0.0014}_{-0.0024}$
$\Delta\Gamma_s$ [ps $^{-1}$ ] (!)	$0.089^{+0.082}_{-0.023}$	$0.089^{+0.090}_{-0.030}$	$0.089^{+0.097}_{-0.036}$
$\Delta\Gamma_s$ [ps $^{-1}$ ]	$0.0811^{+0.0063}_{-0.0062}$	$0.081^{+0.013}_{-0.012}$	$0.081^{+0.019}_{-0.018}$
$B \rightarrow \tau\nu$ [ $10^{-4}$ ] (!)	$1.033^{+0.065}_{-0.345}$	$1.03^{+0.13}_{-0.42}$	$1.03^{+0.20}_{-0.47}$
$B \rightarrow \tau\nu$ [ $10^{-4}$ ]	$1.037^{+0.062}_{-0.155}$	$1.04^{+0.13}_{-0.33}$	$1.04^{+0.19}_{-0.43}$

(!) means that the measurement was not included in the fit.

### 7.4 Pulls

Without  $A_{SL}$ , the  $p$ -value for the 2D SM hypothesis  $\Delta_d = 1$  ( $\Delta_s = 1$ ) is  $0.9 \sigma$  ( $0.3 \sigma$ ), and the 4D SM hypothesis  $\Delta_d = 1 = \Delta_s = 1$  has a  $p$ -value of  $0.7 \sigma$ . With  $A_{SL}$ , the  $p$ -value for the 2D SM hypothesis  $\Delta_d = 1$  ( $\Delta_s = 1$ ) is  $1.2 \sigma$  ( $0.3 \sigma$ ), and the 4D SM hypothesis  $\Delta_d = 1 = \Delta_s = 1$  has a  $p$ -value of  $1.0 \sigma$ .

Quantity(ies)	Without $A_{\text{SL}}$		With $A_{\text{SL}}$	
	Deviation wrt SM	Sc. I	Deviation wrt SM	Sc. I
$\phi_d^\Delta + 2\beta$	1.6 $\sigma$	0.0 $\sigma$	1.6 $\sigma$	0.0 $\sigma$
$\phi_s^\Delta - 2\beta_s$	0.0 $\sigma$	1.1 $\sigma$	0.0 $\sigma$	2.6 $\sigma$
$A_{\text{SL}}$	—	—	2.7 $\sigma$	2.4 $\sigma$
$a_{\text{SL}}^d$	0.4 $\sigma$	0.8 $\sigma$	0.4 $\sigma$	1.1 $\sigma$
$a_{\text{SL}}^s$	1.0 $\sigma$	1.0 $\sigma$	1.0 $\sigma$	1.0 $\sigma$
$\Delta\Gamma_s$	0.3 $\sigma$	0.3 $\sigma$	0.1 $\sigma$	0.1 $\sigma$
$\mathcal{B}(B \rightarrow \tau\nu)$	1.3 $\sigma$	0.8 $\sigma$	1.3 $\sigma$	0.2 $\sigma$
$\mathcal{B}(B \rightarrow \tau\nu), A_{\text{SL}}$	—	—	2.5 $\sigma$	2.1 $\sigma$
$\phi_s^\Delta - 2\beta_s, A_{\text{SL}}$	—	—	2.2 $\sigma$	2.2 $\sigma$
$\mathcal{B}(B \rightarrow \tau\nu), \phi_s^\Delta - 2\beta_s, A_{\text{SL}}$	—	—	2.2 $\sigma$	1.9 $\sigma$

## References

- [1] I.S. Towner and J.C. Hardy, Phys. Rev. **C 79**, 055502 (2009), arXiv:0812.1202 [nucl-ex].
- [2] See sec. 2.
- [3] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38** (2014) 090001.
- [4] Y. Amhis *et al.* [Heavy Flavor Averaging Group (HFAG) Collaboration], arXiv:1412.7515 [hep-ex], and The Heavy Flavor Averaging Group (HFAG), Fall 2014, <http://www.slac.stanford.edu/xorg/hfag>, and references therein.
- [5] For the inclusive average we are taking the BLNP number. (The DGE result is very close to the BLNP result. The uncertainties between BLNP and DGE are hard to compare.) The theoretical error on the inclusive average is obtained by adding linearly the contributions from weak annihilation, subleading shape functions and the HQE uncertainty on  $m_b$ . We use only branching fractions measured for  $B \rightarrow \pi\ell\nu$ . We average the results obtained from the two unquenched Lattice calculations quoted by HFAG [4], adding the systematic uncertainties in such a way that the smallest theoretical error is kept. We keep the smallest theoretical error also for the average between the inclusive and exclusive results.
- [6] We use the educated Rfit approach to average the following values: for  $|V_{ub}|$ ,  $|V_{ub}|_{\text{sl,incl}} = (4.36 \pm 0.18 \pm 0.44) \times 10^{-3}$  and  $|V_{ub}|_{\text{sl,excl}} = (3.28 \pm 0.15 \pm 0.26) \times 10^{-3}$ , and for  $|V_{cb}|$ ,  $|V_{cb}|_{\text{sl,incl}} = (42.42 \pm 0.44 \pm 0.74) \times 10^{-3}$  and  $|V_{cb}|_{\text{sl,excl}} = (38.99 \pm 0.49 \pm 1.17) \times 10^{-3}$ .
- [7] J. P. Lees *et al.*, Phys. Rev. **D 88** 031102 (2013) [arXiv:1207.0698 [hep-ex]],  
 B. Aubert *et al.*, Phys. Rev. **D 81**, 051101(R) (2010),  
 I. Adachi *et al.*, Phys. Rev. Lett. **110**, 131801 (2013), arXiv:1208.4678 [hep-ex],  
 A. Abdesselam *et al.*, arXiv:1409.5269 [hep-ex].

- [8] V. Khachatryan *et al.* [CMS and LHCb Collaborations], arXiv:1411.4413 [hep-ex].
- [9] D. Besson *et al.* [CLEO Collaboration], Phys. Rev. D **80** (2009) 032005 [arXiv:0906.2983 [hep-ex]].  
L. Widhalm *et al.* [Belle Collaboration], Phys. Rev. Lett. **97** (2006) 061804 [hep-ex/0604049].
- [10] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **76** (2007) 052005 [arXiv:0704.0020 [hep-ex]].
- [11] BaBar collaboration, Phys. Rev. D75, 012008 (2007).
- [12] BaBar collaboration, Phys.Rev. D87 (2013) 052009.
- [13] BaBar collaboration, Phys. Rev. D76, 091102 (2007).
- [14] Belle collaboration, Phys. Rev. D87, 031103 (2013).
- [15] Belle collaboration, arXiv:hep-ex/13020551 (2013).
- [16] Belle collaboration, CKM Workshop 2014, Vienna, Austria (2014).
- [17] CDF collaboration, Phys. Rev. Lett. 106, 181802 (2011).
- [18] Cleo collaboration, Phys. Rev. D68, 052002 (2003).
- [19] LHCb, J. High Energ. Phys. 1210, 037 (2012).
- [20] BaBar collaboration, Phys. Rev. D76, 052007 (2007).
- [21] BaBar collaboration, Phys. Rev. Lett. 102, 141802 (2009).
- [22] BaBar collaboration, Phys. Rev. D78, 071104 (2008).
- [23] Belle collaboration, Phys. Rev. Lett. 96, 171801 (2006).
- [24] Belle collaboration, Phys. Rev. D 76, 011104 (2007).
- [25] Belle collaboration, Phys. Rev. Lett. 91, 221801 (2003).
- [26] Belle collaboration, Phys. Rev. D89 072008 (2014).
- [27] Babar collaboration, Phys. Rev. D 88, 012003 (2013).
- [28] Belle collaboration, Phys. Rev. Lett. 98, 221602 (2007).
- [29] P. del Amo Sanchez *et al.* [BaBar Collaboration], Phys. Rev. D **82** (2010) 072004. B. Aubert *et al* [BaBar Collaboration] Phys. Rev. D **78** (2008) 092002. B. Aubert *et al* [BaBar Collaboration], Phys. Rev. D **80** (2009) 092001. K. Trabelsi for the Belle Collaboration, arXiv:1301.2033, preliminary results. R. Aaij *et al* [LHCb Collaboration] Phys.Lett. **B712** (2012) 203.
- [30] R. Aaij *et al* [LHCb Collaboration], arXiv:1407.8136, preliminary results.
- [31] P. del Amo Sanchez *et al* [BaBar Collaboration], Phys.Rev. **D82** (2010) 072006. Y. Horii *et al* [Belle Collaboration], Phys.Rev.Lett. **106** (2011) 231803. K. Trabelsi for the Belle Collaboration, arXiv:1301.2033, preliminary results. R. Aaij *et al* [LHCb Collaboration], Phys.Lett. **B712** (2012) 203. M. Nayak *et al*. [Belle Collaboration], Phys. Rev. D **88** (2013) 9, 091104

- [32] P. del Amo Sanchez *et al* [BaBar Collaboration], Phys. Rev. Lett. **105** (2010) 121801. A. Poluektov *et al* [Belle Collaboration], Phys. Rev. D **81** (2010) 112002. R. Aaij *et al* [LHCb Collaboration], JHEP **10** (2014) 097.
- [33] We take  $\bar{m}_c(m_c) = (1.286 \pm 0.013)$  GeV from an analysis by Kuhn and Steinhauser [36]. We assign an additional theoretical uncertainty of 0.040 GeV in order to take into account: a) an observed difference in analyses with participation of the same authors for the central value when using a somehow different extraction method [34, 35], and b) in order to take into account a likely over-optimistic error range for the gluon condensate in this analysis. This input is consistent with  $\bar{m}_c(m_c) = (1.265 \pm 0.060 \pm 0.050)$  GeV translated from the kinetic charm mass obtained from fits to data from lepton energy and hadronic mass moments in  $B \rightarrow X_c \ell \nu$  decays combined with photon energy moments measured in  $B \rightarrow X_s \gamma$  decays:  $m_c^{kin} = (1.165 \pm 0.050)$  GeV [4].
- [34] I. Allison *et al.* [HPQCD Collaboration], Phys. Rev. D **78** (2008) 054513 [arXiv:0805.2999 [hep-lat]].
- [35] M. Steinhauser, arXiv:0809.1925 [hep-ph].
- [36] J. H. Kuhn, M. Steinhauser and C. Sturm, Nucl. Phys. B **778** (2007) 192 [arXiv:hep-ph/0702103].
- [37] The world average of the top quark mass measurements performed at the Tevatron and LHC [38] interpreted as a pole mass is translated into  $m_t(m_t)$  in the  $\overline{\text{MS}}$  at 1-loop order.
- [38] [ATLAS and CDF and CMS and D0 Collaborations], arXiv:1403.4427 [hep-ex].
- [39] A. J. Buras and D. Guadagnoli, Phys. Rev. D **78** (2008) 033005 [arXiv:0805.3887 [hep-ph]].  
A. J. Buras, D. Guadagnoli and G. Isidori, Phys. Lett. B **688** (2010) 309 [arXiv:1002.3612 [hep-ph]].
- [40] A. Lenz, U. Nierste, J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker and S. Monteil *et al.*, Phys. Rev. D **83** (2011) 036004 [arXiv:1008.1593 [hep-ph]].
- [41] J. Brod and M. Gorbahn, Phys. Rev. Lett. **108** (2012) 121801 [arXiv:1108.2036 [hep-ph]].
- [42] J. Brod and M. Gorbahn, Phys. Rev. D **82** (2010) 094026 [arXiv:1007.0684 [hep-ph]].
- [43] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347** (1990) 491.  
S. Herrlich and U. Nierste, Nucl. Phys. **B 419**, 292 (1994).
- [44] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
- [45] A. Lenz, private communication (2010).
- [46] A. Lenz, U. Nierste, J. Charles, S. Descotes-Genon, H. Lacker, S. Monteil, V. Niess and S. T'Jampens, Phys. Rev. D **86** (2012) 033008 [arXiv:1203.0238 [hep-ph]].
- [47] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **86** (2012) 072009 [arXiv:1208.5813 [hep-ex]].
- [48] J. P. Lees *et al.* [BaBar Collaboration], arXiv:1305.1575 [hep-ex].

- [49] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **82** (2010) 012003 [Erratum-ibid. D **83** (2011) 119901] [arXiv:0904.3907 [hep-ex]].
- [50] The LHCb Collaboration, LHCb-CONF-2012-022.
- [51] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **84** (2011) 052007 [arXiv:1106.6308 [hep-ex]].
- [52] The CDF Collaboration, <http://www-cdf.fnal.gov/physics/new/bottom/070816.blessed-acp-bsemil/>
- [53] S. Aoki, Y. Aoki, C. Bernard, T. Blum, G. Colangelo, M. Della Morte, S. Drr and A. X. El Khadra *et al.*, arXiv:1310.8555 [hep-lat].
- [54] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **89** (2014) 1, 012002 [arXiv:1310.0447 [hep-ex]].
- [55] G. Borissov and B. Hoeneisen, Phys. Rev. D **87** (2013) 7, 074020 [arXiv:1303.0175 [hep-ex]].
- [56] U. Nierste in *Effect of Delta Gamma on the dimuon asymmetry in B decays*, 8th International Workshop on the CKM Unitarity Triangle (CKM2014), 8-12 Sep. 2014, Vienna, Austria).
- [57] R. Aaij *et al.* [LHCb Collaboration], arXiv:1409.8586 [hep-ex].
- [58] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **110** (2013) 011801 [arXiv:1207.1769 [hep-ex]].
- [59] R. Aaij *et al.* [LHCb Collaboration], Phys. Lett. B **728** (2014) 607 [arXiv:1308.1048 [hep-ex]].
- [60] B. Blossier *et al.*, JHEP **0907** (2009) 043 [arXiv:0904.0954 [hep-lat]].
- [61] E. Follana, C. T. H. Davies, G. P. Lepage and J. Shigemitsu [HPQCD Collaboration and UKQCD Collaboration], Phys. Rev. Lett. **100**, 062002 (2008) [arXiv:0706.1726 [hep-lat]].
- [62] A. Bazavov *et al.* [MILC Collaboration], PoS LATTICE **2010** (2010) 074 [arXiv:1012.0868 [hep-lat]].
- [63] J. Laiho and R. S. Van de Water, PoS LATTICE **2011** (2011) 293 [arXiv:1112.4861 [hep-lat]].
- [64] R. Arthur *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **87** (2013) 094514 [arXiv:1208.4412 [hep-lat]].
- [65] R. J. Dowdall, C. T. H. Davies, G. P. Lepage and C. McNeile, arXiv:1303.1670 [hep-lat].
- [66] P. Dimopoulos, R. Frezzotti, P. Lami, V. Lubicz, E. Picca, L. Riggio, G. C. Rossi and F. Sanfilippo *et al.*, arXiv:1311.3080 [hep-lat].
- [67] S. Durr *et al.*, Phys. Rev. D **81** (2010) 054507 [arXiv:1001.4692 [hep-lat]].
- [68] A. Bazavov, C. Bernard, C. DeTar, J. Foley, W. Freeman, S. Gottlieb, U. M. Heller and J. E. Hetrick *et al.*, arXiv:1301.5855 [hep-ph].
- [69] C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na and J. Shigemitsu, Phys. Rev. D **82** (2010) 114504 [arXiv:1008.4018 [hep-lat]].

- [70] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaboration], Phys. Rev. D **85** (2012) 114506 [arXiv:1112.3051 [hep-lat]].
- [71] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaborations], PoS LATTICE **2012** (2012) 159 [arXiv:1210.8431 [hep-lat]].
- [72] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **86** (2012) 054510 [arXiv:1206.4936 [hep-lat]].
- [73] N. Carrasco, M. Ciuchini, P. Dimopoulos, R. Frezzotti, V. Gimenez, G. Herdoiza, V. Lubicz and C. Michael *et al.*, arXiv:1308.1851 [hep-lat].
- [74] F. Bernardoni *et al.* [ALPHA Collaboration], Phys. Lett. B **735** (2014) 349 [arXiv:1404.3590 [hep-lat]].
- [75] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel and G. P. Lepage, Phys. Rev. D **85** (2012) 031503 [arXiv:1110.4510 [hep-lat]].
- [76] H. Na, C. J. Monahan, C. T. H. Davies, R. Horgan, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **86** (2012) 034506 [arXiv:1202.4914 [hep-lat]].
- [77] R. J. Dowdall *et al.* [HPQCD Collaboration], arXiv:1302.2644 [hep-lat].
- [78] V. Lubicz, F. Mescia, S. Simula, C. Tarantino and f. t. E. Collaboration, Phys. Rev. D **80** (2009) 111502 [arXiv:0906.4728 [hep-lat]].
- [79] A. Bazavov, C. Bernard, C. M. Bouchard, C. DeTar, D. Du, A. X. El-Khadra, J. Foley and E. D. Freeland *et al.*, arXiv:1212.4993 [hep-lat].
- [80] P. A. Boyle, J. M. Flynn, N. Garron, A. Jttner, C. T. Sachrajda, K. Sivalingam and J. M. Zanotti, JHEP **1308** (2013) 132 [arXiv:1305.7217 [hep-lat]].
- [81] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86** (2012) 010001.
- [82] S. Di Vita, B. Haas, V. Lubicz, F. Mescia, S. Simula *et al.*, PoS **LAT2010** (2010) . [arXiv:1104.0869 [hep-lat]].
- [83] H. Na, C. T. H. Davies, E. Follana, J. Koponen, G. P. Lepage and J. Shigemitsu, Phys-RevD.84.114505,2011 [arXiv:1109.1501 [hep-lat]].
- [84] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **82**, 114506 (2010) [arXiv:1008.4562 [hep-lat]].
- [85] M. Constantinou *et al.* [ETM Collaboration], Phys. Rev. D **83** (2011) 014505 [arXiv:1009.5606 [hep-lat]].
- [86] S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch and T. Lippert *et al.*, Phys. Lett. B **705** (2011) 477 [arXiv:1106.3230 [hep-lat]].
- [87] T. Bae *et al.* [SWME Collaboration], arXiv:1402.0048 [hep-lat].
- [88] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Phys. Rev. D **80**, 014503 (2009) [arXiv:0902.1815 [hep-lat]].

- [89] A. Bazavov, C. Bernard, C. M. Bouchard, C. DeTar, M. Di Pierro, A. X. El-Khadra, R. T. Evans and E. D. Freeland *et al.*, Phys. Rev. D **86** (2012) 034503 [arXiv:1205.7013 [hep-lat]].