UPDATED RESULTS ON THE CKM MATRIX

Including results presented up to
EPS 2015

Preliminary

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The CKMfitter Group

Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach, using the *R*fit model to treat theoretical uncertainties. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

CP VIOLATION AND THE CKM MATRIX:
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES
By CKMfitter Group
Eur. Phys. J. **C41**, 1-131, 2005 [hep-ph/0406184]

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http://ckmfitter.in2p3.fr

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1 Inputs

Parameter	Value ± Error(s)	Reference	Errors GS	Errors TH
$\vert V_{ud}\vert$ (nuclei)	$0.97425 \pm 0 \pm 0.00022$	[1]	-	$\star$
$\vert V_{us}\vert f_+^{K\to\pi}(0)$	0.2163 ± 0.0005	[3]	$\star$	$\star$
$\vert V_{cd}\vert$ (νN)	0.230 ± 0.011	[3]	$\star$	-
$\vert V_{cs}\vert$ $(W \to c\bar{s})$	$0.94^{+0.32}_{-0.26} \pm 0.13$	[3]	$\star$	$\star$
$\vert V_{ub}\vert$ (semileptonic)	$(4.01 \pm 0.08 \pm 0.22) \times 10^{-3}$	[4–6]	$\star$	$\star$
$\vert V_{cb}\vert$ (semileptonic)	$(41.00 \pm 0.33 \pm 0.74) \times 10^{-3}$	[4, 6]	$\star$	$\star$
$\mathcal{B}(\Lambda_p \to p\mu^-\overline{\nu}_\mu)_{q^2>15}/\mathcal{B}(\Lambda_p \to \Lambda_c\mu^-\overline{\nu}_\mu)_{q^2>7}$	$(1.00 \pm 0.09) \times 10^{-2}$	[7]	$\star$	-
$\mathcal{B}(B^- \to \tau^-\overline{\nu}_\tau)$	$(1.08 \pm 0.21) \times 10^{-4}$	[4, 8]	$\star$	-
$\mathcal{B}(D_s^- \to \mu^-\overline{\nu}_\mu)$	$(5.57 \pm 0.24) \times 10^{-3}$	[4]	$\star$	-
$\mathcal{B}(D_s^- \to \tau^-\overline{\nu}_\tau)$	$(5.55 \pm 0.24) \times 10^{-2}$	[4]	$\star$	-
$\mathcal{B}(D^- \to \mu^-\overline{\nu}_\mu)$	$(3.74 \pm 0.17) \times 10^{-4}$	[4]	$\star$	$\star$
$\mathcal{B}(K^- \to e^-\overline{\nu}_e)$	$(1.581 \pm 0.008) \times 10^{-5}$	[3]	$\star$	-
$\mathcal{B}(K^- \to \mu^-\overline{\nu}_\mu)$	0.6355 ± 0.0011	[3]	$\star$	-
$\mathcal{B}(\tau^- \to K^-\overline{\nu}_\tau)$	$(0.6955 \pm 0.0096) \times 10^{-2}$	[4]	$\star$	-
$\mathcal{B}(K^- \to \mu^-\overline{\nu}_\mu)/\mathcal{B}(\pi^- \to \mu^-\overline{\nu}_\mu)$	1.3365 ± 0.0032	[3]	$\star$	-
$\mathcal{B}(\tau^- \to K^-\overline{\nu}_\tau)/\mathcal{B}(\tau^- \to \pi^-\overline{\nu}_\tau)$	$(6.431 \pm 0.094) \times 10^{-2}$	[4]	$\star$	-
$\mathcal{B}(B_s \to \mu\mu)$	$(2.8^{+0.7}_{-0.6}) \times 10^{-9}$	[9]	$\star$	-
$\vert V_{cd}\vert f_+^{D\to\pi}(0)$	0.148 ± 0.004	[10]	$\star$	-
$\vert V_{cs}\vert f_+^{D\to K}(0)$	0.712 ± 0.007	[10, 11]	$\star$	-
$\vert\varepsilon_K\vert$	$(2.228 \pm 0.011) \times 10^{-3}$	[3]	$\star$	-
Δm_d	$(0.510 \pm 0.003)\ \mathrm{ps}^{-1}$	[4]	$\star$	-
Δm_s	$(17.757 \pm 0.021)\ \mathrm{ps}^{-1}$	[4]	$\star$	-
$\sin(2\beta)_{[c\bar{c}]}$	0.691 ± 0.017	[4]	$\star$	-
$(\phi_s)_{[b\to c\bar{s}s]}$	-0.015 ± 0.035	[4]	$\star$	-
$S_{\pi\pi}^{+-}$, $C_{\pi\pi}^{+-}$, $C_{\pi\pi}^{00}$,$\mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis	[12–20]	$\star$	-
$S_{\rho\rho,L}^{+-}$, $C_{\rho\rho,L}^{+-}$, $S_{\rho\rho}^{00}$, $C_{\rho\rho}^{00}$,$\mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[21–27]	$\star$	-
$B^0 \to (\rho\pi)^0 \to 3\pi$	Time-dependent Dalitz analysis	[28, 29]	$\star$	-
$B^- \to D^{(*)}K^{(*)-}$	Inputs to GLW analysis	[30, 31]	$\star$	-
$B^- \to D^{(*)}K^{(*)-}$	Inputs to ADS analysis	[31, 32]	$\star$	-
$B^- \to D^{(*)}K^{(*)-}$	GGSZ Dalitz analysis	[33]	$\star$	-

Table 1: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate* R*fit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the "GS" column by a star; measurements or parameters that have systematic theoretical errors are marked in the "TH" column by a star. Upper part: experimental determinations of the CKM matrix elements. Lower part: CP-violation and mixing observables.*

Parameter	Value ± Error(s)	Reference	Errors GS	Errors TH
$\overline{m}_c(m_c)$	$(1.286 \pm 0.013 \pm 0.040)$ GeV	[34]	$\star$	$\star$
$\overline{m}_t(m_t)$	$(165.95 \pm 0.35 \pm 0.64)$ GeV	[38]	$\star$	$\star$
$\alpha_s(m_Z)$	$0.1185 \pm 0 \pm 0.0006$	[3]	-	$\star$
B_K	$0.7615 \pm 0.0027 \pm 0.0137$	[2]	$\star$	$\star$
κ_ϵ	$0.940 \pm 0.013 \pm 0.023$	[40, 41]	$\star$	$\star$
η_{cc}	$1.87 \pm 0 \pm 0.76$	[42]	-	$\star$
η_{ct}	$0.497 \pm 0 \pm 0.047$	[43]	-	$\star$
η_{tt}	$0.5765 \pm 0 \pm 0.0065$	[44]	-	$\star$
$\eta_B(\overline{\mathrm{MS}})$	$0.5510 \pm 0 \pm 0.0022$	[45, 46]	-	$\star$
f_{B_s}	$(224.0 \pm 1.0 \pm 2.0)$ MeV	[2]	$\star$	$\star$
B_s	$1.320 \pm 0.016 \pm 0.030$	[2]	$\star$	$\star$
f_{B_s}/f_{B_d}	$1.205 \pm 0.003 \pm 0.006$	[2]	$\star$	$\star$
B_s/B_d	$1.023 \pm 0.013 \pm 0.014$	[2]	$\star$	$\star$
f_K	$(155.2 \pm 0.2 \pm 0.6)$ MeV	[2]	$\star$	$\star$
f_K/f_π	$1.1952 \pm 0.0007 \pm 0.0029$	[2]	$\star$	$\star$
f_{D_s}	$(248.2 \pm 0.3 \pm 1.9)$ MeV	[2]	$\star$	$\star$
f_{D_s}/f_D	$1.175 \pm 0.001 \pm 0.004$	[2]	$\star$	$\star$
$f_+^{K\to\pi}(0)$	$0.9645 \pm 0.0015 \pm 0.0045$	[2]	$\star$	$\star$
$f_+^{D\to\pi}(0)$	$0.666 \pm 0.020 \pm 0.048$	[2]	$\star$	$\star$
$f_+^{D\to K}(0)$	$0.747 \pm 0.011 \pm 0.034$	[2]	$\star$	$\star$
$\zeta(\Lambda_p \to p\mu^-\overline{\nu}_\mu)_{q^2>15}/\zeta(\Lambda_p \to \Lambda_c\mu^-\overline{\nu}_\mu)_{q^2>7}$	$1.471 \pm 0.096 \pm 0.290$	[2]	$\star$	$\star$

Table 2: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate* R*fit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the "GS" column by a star; measurements or parameters that have systematic theoretical errors are marked in the "TH" column by a star. Upper part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

2 Lattice QCD averages

Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

2.1 Method of averaging

We collect the relevant calculations of the quantity that we are interested in: we take only unquenched results with 2 or 2+1 dynamical fermions, even those from proceedings without a companion article. For the calculations published before the end of November 2013, we have followed the classification of the Flavour Lattice Averaging Group [54] and kept only results with green squares. However, we stress that we perform our averages in a different manner from FLAG.

In these results, we separate the error estimates into a Gaussian part and a flat part that is treated à la Rfit. The Gaussian part collects the uncertainties from purely statistical origin, but also the systematics that can be controlled and treated in a similar way (e.g., interpolation or fitting in some cases). The remaining systematics constitute the Rfit error. If there are several sources of error in the Rfit category, we add them linearly [1].

The Rfit model is simple but also very strict. It amounts to assuming that the theoretical uncertainty is rigorously constrained by a mathematical bound that is our only piece of information. If Rfit is taken stricto sensu and the individual likelihoods are combined in the usual way (by multiplication), the final uncertainty can be underestimated, in particular in the case of marginally compatible values.

We correct this effect by adopting the following averaging recipe. The central value is obtained by combining the whole likelihoods. Then we combine the Gaussian uncertainties by combining likelihoods restricted to their Gaussian part. Finally we assign to this combination the smallest of the individual Rfit uncertainties. The underlying idea is twofold:

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
- this best estimate should not be penalized by less precise methods (as it would happen be the case if one would take the dispersion of the individual central values as a guess of the combined theoretical uncertainty).

It should be stressed that the concept of a theoretical uncertainty is ill-defined, and the combination of them even more. Thus our approach is only one among the alternatives that can be found in the literature. In contrast to some of the latter, ours is algorithmic and can be reproduced.

2.2 Decay constants

2.2.1 Light mesons

f_K

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[1] keeping in mind that in many papers in the literature, this combination is done in quadrature and the splitting between different sources is not published.

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[61]	2	158.1	0.8	3.1
HPQCD07	[62]	2+1	157	0.6	3.3
MILC10	[63]	2+1	156.1	0.4	$^{+0.6}_{-0.9}$
LVdW11	[64]	2+1	156.8	1.0	3.3
RBC-UKQCD12	[65]	2+1	152.4	3.0	2.2
HPQCD13	[66]	2+1+1	155.4	0.2	0.6
ETMC14	[67]	2+1+1	155.0	1.4	2.0
Our average			155.2	0.2	0.6

f_K/f_π

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[61]	2	1.210	0.006	0.024
HPQCD/UKQCD07	[62]	2+1	1.189	0.002	0.014
MILC10	[63]	2+1	1.197	0.002	$^{+0.003}_{-0.007}$
BMW10	[68]	2+1	1.192	0.007	0.013
LVdW11	[64]	2+1	1.202	0.011	0.024
RBC-UKQCD12	[65]	2+1	1.1991	0.0116	0.0185
HPQCD13	[66]	2+1+1	1.1938	0.0015	0.0032
FNAL-MILC14	[69]	2+1+1	1.1956	0.0010	$^{+0.0033}_{-0.0024}$
ETMC14	[67]	2+1+1	1.188	0.011	0.020
Our average			1.1952	0.0007	0.0029

Results have been corrected to express results in terms of the decay constants defined in QCD (electromagnetic corrections are applied at the level of the branching ratios).

2.2.2 Charmed mesons

f_{D_s}

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[61]	2	244	3	9
HPQCD10	[71]	2+1	248.0	1.4	4.5
FNAL-MILC11	[70]	2+1	260.1	8.9	16.2
FNAL-MILC14	[69]	2+1+1	249.0	0.3	$^{+1.7}_{-2.1}$
ETMC14	[67]	2+1+1	247.2	3.9	2.2
Our average			248.2	0.3	1.9

f_{D_s}/f_D

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[61]	2	1.24	0.03	0.01
FNAL-MILC11	[70]	2+1	1.188	0.014	0.054
HPQCD12	[72]	2+1	1.187	0.004	0.023
FNAL-MILC14	[69]	2+1+1	1.1712	0.0010	$^{+0.0037}_{-0.0040}$
ETMC14	[67]	2+1+1	1.192	0.019	0.017
Our average			1.175	0.001	0.004

2.2.3 Beauty mesons

f_{B_s}

Reference	Article	N_f	Mean	Stat	Syst
ETMC13	[73]	2	228	5	9
ALPHA14	[74]	2	224	14	2
HPQCD11	[75]	2+1	225.0	2.9	5.4
FNAL-MILC11	[70]	2+1	242.0	5.1	21.2
HPQCD12	[76]	2+1	228.0	1.4	17.5
RBC-UKQCD14	[77]	2+1	235.4	5.2	28.1
HPQCD13	[78]	2+1+1	224.0	2.5	7.2
Our average			224	1.0	2.0

f_{B_s}/f_B

Reference	Article	N_f	Mean	Stat	Syst
ETMC13	[73]	2	1.206	0.010	0.026
ALPHA14	[74]	2	1.203	0.062	0.019
FNAL-MILC11	[70]	2+1	1.229	0.013	0.046
HPQCD12	[76]	2+1	1.188	0.012	0.025
RBC-UKQCD14	[77]	2+1	1.223	0.013	0.106
HPQCD13	[78]	2+1+1	1.205	0.004	0.007
Our average			1.205	0.003	0.006

2.3 Semileptonic form factors

2.3.1 $K \to \pi \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[79]	2	0.9560	0.0057	0.0118
MILC12	[80]	2+1	0.9667	0.0023	0.0055
RBC-UKQCD13	[81]	2+1	0.9670	0.0020	$^{+0.024}_{-0.066}$
Our average			0.9645	0.0015	0.0045

2.3.2 $D \to \pi \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
ETMC11	[82]	2	0.65	0.06	0.12
HPQCD11	[83]	2+1	0.666	0.021	0.048
Our average			0.666	0.020	0.048

2.3.3 $D \to K\ell\nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
ETMC11	[82]	2	0.76	0.05	0.11
HPQCD10	[84]	2+1	0.747	0.011	0.034
Our average			0.747	0.011	0.034

2.3.4 $\Lambda_b \to p\mu^-\bar{\nu}_\mu$ and $\Lambda_b \to \Lambda_c\bar{\nu}_\mu$

$\zeta(\Lambda_b \to p\mu^-\bar{\nu}_\mu)_{q^2>15}/\zeta(\Lambda_b \to \Lambda_c\mu^-\bar{\nu}_\mu)_{q^2>7}$

Reference	Article	N_f	Mean	Stat	Syst
DLM15	[85]	2+1	1.471	0.096	0.290
Our average			1.471	0.096	0.290

2.4 Meson mixing

2.4.1 Kaon mixing

$B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$

Reference	Article	N_f	Mean	Stat	Syst
ETMC 10	[86]	2	0.532	0.019	0.026
LVdW11	[64]	2+1	0.5572	0.0028	0.0257
BMW11	[87]	2+1	0.5644	0.0059	0.0100
RBC-UKQCD12	[65]	2+1	0.554	0.008	0.022
SWME14	[88]	2+1	0.5388	0.0034	0.0442
Our average for $B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$			0.5562	0.0020	0.0100
Our average for $\hat{B}_K$			0.7615	0.0027	0.0137

2.4.2 $B_{d,s}$ mixing

$\hat{B}_{B_s}$

Reference	Article	N_f	Mean	Stat	Syst
ETMC13	[73]	2	1.32	0.04	0.03
HPQCD09	[89]	2+1	1.326	0.018	0.040
Our average			1.320	0.016	0.030

Ref. [89] provide only f_{B_s} and $f_{B_s}\sqrt{\hat{B}_{B_s}}$, and we assumed that the systematics were completely correlated to extract $\hat{B}_{B_s}$.

$\hat{B}_{B_s}/\hat{B}_{B_d}$

Reference	Article	N_f	Mean	Stat	Syst
ETMC13	[73]	2	1.007	0.015	0.014
HPQCD09	[89]	2+1	1.053	0.025	0.023
FNAL/MILC12	[90]	2+1	1.064	0.076	0.193
Our average			1.023	0.013	0.014

Refs. [89] and [90] provide only ξ and f_{B_s}/f_{B_d}. For Refs. [89], we have extracted $\hat{B}_{B_s}/\hat{B}_{B_d}$ in both cases assuming a total correlation in the systematics of ξ and $\hat{B}_{B_s}/\hat{B}_{B_d}$. For Ref. [90], we have considered all uncertainties as uncorrelated, as the studies of the decay constants and the bag parameters have been performed in different settings, with different categories of systematics.

2.5 $|V_{ub}|$

The direct determination of $|V_{ub}|$ has been determined in several distinct approaches, namely with B decays in leptonic, exclusive semileptonic, and inclusive semileptonic modes, with data primarily from the $B-$ factories. A precise measurement of the ratio $|V_{ub}|/|V_{cb}|$ has also recently been performed using exclusive semileptonic Λ_b decays with data from LHCb. Here we describe the approach for the treatment of the theoretical uncertainties on $|V_{ub}|/|V_{cb}|$ and $|V_{ub}|$ for use in CKMFitter, and the combination of the exclusive and inclusive semileptonic B decay measurements. The leptonic B decay uncertainty is described elsewhere. For consistent treatment of theoretical uncertainties across CKMFitter input, we take the linear sum of theoretical uncertainties (apart from LQCD statistical errors), rather than the usual quadratic sum approach given by HFAG and theoretical papers.

2.5.1 Semileptonic B decays

For the inclusive semileptonic B decay approach, we take the current $|V_{ub}|$ results from HFAG [4] with theory from BLNP [91], $|V_{ub}| = (4.45 \pm 0.16(\text{exp.})^{+0.21}_{-0.22}(\text{theory.})) \times 10^{-3}$ (similar values are also found by GGOU [92], $|V_{ub}| = (4.51 \pm 0.16(\text{exp.})^{+0.12}_{-0.15}(\text{theory}))$. With the quoted error profile from HFAG, we form the quadratic sum of all experimental uncertainties, as well as the uncertainty on m_b derived from an independent fit to measured moments of $b \to c\ell\nu$ semileptonic decays, and form a linear sum of the theoretical uncertainties, including the kinetic to shape-function scheme translation uncertainty on m_b [91]. The inclusive approach yields $|V_{ub}| = (4.45 \pm 0.18(\text{exp.}) \pm 0.31(\text{theory})) \times 10^{-3}$.

For the exclusive semileptonic B decay approach, we take the latest results from [93], which includes an update to the $B \to \pi$ form factors. They perform a fit to LQCD calculations and all available $B \to \pi\ell\nu$ q^2-binned experimental data, finding a result that is slightly higher than those from previous calculations: $|V_{ub}| = (3.72 \pm 0.16(\text{fit})) \times 10^{-3}$. The single quoted uncertainty here is due to the nature of the fit method. The split between the theory and experimental uncertainties is estimated to be $|V_{ub}| = (3.72 \pm 0.09(\text{exp.}) \pm 0.13(\text{LQCD})) \times 10^{-3}$, derived from the LQCD error profile in Ref. [93]. The LQCD uncertainties are added linearly according to the provided systematic error profile for the form factors at $q^2 = 20$ GeV, which is representative of the uncertainties in the full binned fit. In Ref. [93] the LQCD statistical uncertainty could not be isolated from the systematic uncertainties, and while it means it could not be excluded from the linear sum, it had already been combined in quadrature with some other systematic uncertainties (the effects should balance in the final error budget). This treatment yields $|V_{ub}| = (3.72 \pm 0.09(\text{exp.}) \pm 0.22(\text{LQCD})) \times 10^{-3}$ for the exclusive approach.

We combine the exclusive and inclusive semileptonic B determinations using the educated Rfit approach to yield $|V_{ub}|_{\mathrm{SL}} = (4.01 \pm 0.08(\mathrm{exp.}) \pm 0.22(\mathrm{LQCD})) \times 10^{-3}$.

2.5.2 Semileptonic Λ_b decays

In 2015, a third semileptonic decay approach was demonstrated with a measurement of $(\Lambda_b \to p\mu\nu)/\mathcal{B}(\Lambda_b \to \Lambda_c\mu\nu)$ at high values of q^2 [7], in conjunction with a new LQCD calculation that relates the measurement to $|V_{ub}|/|V_{cb}|$ [85]. The latter derives the ratio of partially integrated decay rates to be $\zeta_{p\mu\nu}/\zeta_{\Lambda_c\mu\nu} = 1.471 \pm 0.095(\mathrm{LQCD\ stat.}) \pm 0.109(\mathrm{LQCD\ sys.})$, determined from a global fit to LQCD data. To determine the linear LQCD systematic uncertainty sum, we take the uncertainty compositions at $q^2 = 15$ and $7\ \mathrm{GeV}^2/c^2$ for $\Lambda_b \to p\mu\nu$ and $\Lambda_b \to \Lambda_c\mu\nu$ respectively, and find that $\zeta_{p\mu\nu}/\zeta_{\Lambda_c\mu\nu} = 1.471 \pm 0.095(\mathrm{LQCD\ stat.}) \pm 0.291(\mathrm{LQCD\ sys.})$. We directly use the ratio in the global CKM fit, and therefore do not provide a combination. The Λ_b approach yields $|V_{ub}|/|V_{cb}| = (0.083 \pm 0.004(\mathrm{exp.}) \pm 0.003(\mathrm{LQCD\ stat.}) \pm 0.008(\mathrm{LQCD\ sys.})) \times 10^{-3}$.

3 Results

Observable	**central** $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
A	$0.8227^{+0.0066}_{-0.0136}$	$0.823^{+0.013}_{-0.027}$	$0.823^{+0.020}_{-0.035}$
λ	$0.22543^{+0.00042}_{-0.00031}$	$0.22543^{+0.00075}_{-0.00064}$	$0.22543^{+0.00101}_{-0.00097}$
$\bar{\rho}$	$0.1504^{+0.0121}_{-0.0062}$	$0.150^{+0.029}_{-0.013}$	$0.150^{+0.037}_{-0.019}$
$\bar{\eta}$	$0.3540^{+0.0069}_{-0.0076}$	$0.354^{+0.016}_{-0.019}$	$0.354^{+0.025}_{-0.027}$
$J\ \ [10^{-5}]$	$3.140^{+0.069}_{-0.084}$	$3.14^{+0.16}_{-0.21}$	$3.14^{+0.26}_{-0.31}$
$\sin 2\alpha$	$-0.013^{+0.034}_{-0.071}$	$-0.013^{+0.069}_{-0.168}$	$-0.01^{+0.11}_{-0.22}$
$\sin 2\alpha$ (!)	$-0.024^{+0.038}_{-0.134}$	$-0.024^{+0.075}_{-0.181}$	$-0.02^{+0.11}_{-0.23}$
$\sin 2\beta$	$0.710^{+0.011}_{-0.011}$	$0.710^{+0.025}_{-0.021}$	$0.710^{+0.039}_{-0.032}$
$\sin 2\beta$ (!)	$0.748^{+0.030}_{-0.032}$	$0.748^{+0.056}_{-0.050}$	$0.748^{+0.071}_{-0.065}$
$\alpha\ [^\circ]$	$90.4^{+2.0}_{-1.0}$	$90.4^{+4.8}_{-2.0}$	$90.4^{+6.2}_{-3.1}$
$\alpha\ [^\circ]$ (!)	$90.7^{+3.9}_{-1.1}$	$90.7^{+5.2}_{-2.1}$	$90.7^{+6.5}_{-3.2}$
$\alpha\ [^\circ]$ (dir. meas.)	$87.6^{+3.5}_{-3.3} \| -1.1^{+3.8}_{-3.9}$	$87.6^{+10.2}_{-6.5} \| -1.1^{+7.9}_{-9.1}$	$87.6^{+16.3}_{-9.8} \| -1^{+12}_{-15}$
$\beta\ [^\circ]$	$22.62^{+0.44}_{-0.42}$	$22.62^{+1.03}_{-0.84}$	$22.6^{+1.7}_{-1.3}$
$\beta\ [^\circ]$ (!)	$24.3^{+1.3}_{-1.4}$	$24.3^{+2.5}_{-2.1}$	$24.3^{+3.2}_{-2.7}$
$\beta\ [^\circ]$ (dir. meas.)	$21.85^{+0.68}_{-0.67}$	$21.9^{+1.4}_{-1.3}$	$21.9^{+2.1}_{-2.0}$
$\gamma\ [^\circ]$	$67.01^{+0.88}_{-1.99}$	$67.0^{+1.8}_{-4.6}$	$67.0^{+2.8}_{-5.8}$
$\gamma\ [^\circ]$ (!)	$66.85^{+0.94}_{-3.44}$	$66.9^{+1.9}_{-4.9}$	$66.9^{+2.8}_{-6.0}$
$\gamma\ [^\circ]$ (dir. meas.)	$73.2^{+6.3}_{-7.0}$	73^{+13}_{-15}	73^{+20}_{-24}
R_u	$0.3847^{+0.0070}_{-0.0068}$	$0.385^{+0.016}_{-0.014}$	$0.385^{+0.027}_{-0.020}$
R_t	$0.9206^{+0.0059}_{-0.0133}$	$0.921^{+0.012}_{-0.031}$	$0.921^{+0.019}_{-0.039}$
$\bar{\rho}_s$	$-0.00805^{+0.00034}_{-0.00065}$	$-0.00805^{+0.00069}_{-0.00154}$	$-0.0080^{+0.0010}_{-0.0020}$
$\bar{\eta}_s$	$-0.01897^{+0.00041}_{-0.00036}$	$-0.01897^{+0.00101}_{-0.00084}$	$-0.0190^{+0.0014}_{-0.0013}$
$\beta_s \equiv -\arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right)$ [rad]	$0.01882^{+0.00036}_{-0.00042}$	$0.01882^{+0.00083}_{-0.00102}$	$0.0188^{+0.0013}_{-0.0014}$
$\sin 2\beta_s$	$0.03761^{+0.00073}_{-0.00082}$	$0.0376^{+0.0017}_{-0.0020}$	$0.0376^{+0.0027}_{-0.0029}$

(!) means that the quantity was not included in the fit, || indicates the union of the confidence intervals considered.

Observable	**central** $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
$\lvert V_{ud}\rvert$	$0.974254^{+0.000071}_{-0.000097}$	$0.97425^{+0.00015}_{-0.00018}$	$0.97425^{+0.00022}_{-0.00023}$
$\lvert V_{us}\rvert$	$0.22542^{+0.00042}_{-0.00031}$	$0.22542^{+0.00076}_{-0.00065}$	$0.22542^{+0.00101}_{-0.00097}$
$\lvert V_{ub}\rvert$	$0.003714^{+0.000072}_{-0.000060}$	$0.00371^{+0.00017}_{-0.00013}$	$0.00371^{+0.00027}_{-0.00020}$
$\lvert V_{cd}\rvert$	$0.22529^{+0.00041}_{-0.00032}$	$0.22529^{+0.00075}_{-0.00065}$	$0.22529^{+0.00100}_{-0.00098}$
$\lvert V_{cs}\rvert$	$0.973394^{+0.000074}_{-0.000096}$	$0.97339^{+0.00016}_{-0.00018}$	$0.97339^{+0.00024}_{-0.00024}$
$\lvert V_{cb}\rvert$	$0.04180^{+0.00033}_{-0.00068}$	$0.04180^{+0.00065}_{-0.00130}$	$0.04180^{+0.00097}_{-0.00164}$
$\lvert V_{td}\rvert$	$0.008676^{+0.000087}_{-0.000150}$	$0.00868^{+0.00018}_{-0.00044}$	$0.00868^{+0.00027}_{-0.00058}$
$\lvert V_{ts}\rvert$	$0.04107^{+0.00031}_{-0.00067}$	$0.04107^{+0.00063}_{-0.00128}$	$0.04107^{+0.00094}_{-0.00161}$
$\lvert V_{tb}\rvert$	$0.999118^{+0.000024}_{-0.000014}$	$0.999118^{+0.000054}_{-0.000027}$	$0.999118^{+0.000068}_{-0.000041}$
$\lvert V_{ud}\rvert$ (!)	$0.974255^{+0.000072}_{-0.000096}$	$0.97425^{+0.00015}_{-0.00018}$	$0.97425^{+0.00023}_{-0.00024}$
$\lvert V_{us}\rvert$ (!)	$0.224486^{+0.001130}_{-0.000067}$	$0.22449^{+0.00200}_{-0.00013}$	$0.22449^{+0.00208}_{-0.00020}$
$\lvert V_{ub}\rvert$ (!)	$0.00357^{+0.00015}_{-0.00014}$	$0.00357^{+0.00026}_{-0.00024}$	$0.00357^{+0.00037}_{-0.00032}$
$\lvert V_{cb}\rvert$ (!)	$0.04299^{+0.00036}_{-0.00141}$	$0.04299^{+0.00070}_{-0.00241}$	$0.0430^{+0.0011}_{-0.0028}$
Δm_d [ps^{-1}] (!)	$0.555^{+0.043}_{-0.035}$	$0.555^{+0.070}_{-0.072}$	$0.555^{+0.099}_{-0.112}$
Δm_s [ps^{-1}] (!)	$16.73^{+0.82}_{-0.57}$	$16.7^{+2.0}_{-1.3}$	$16.7^{+2.6}_{-1.8}$
$\lvert\epsilon_K\rvert$ [10^{-3}] (!)	$2.20^{+0.47}_{-0.49}$	$2.20^{+0.57}_{-0.61}$	$2.20^{+0.68}_{-0.69}$
m_t [GeV/c^2] (!)	$161.5^{+6.9}_{-2.2}$	$161.5^{+14.9}_{-4.4}$	$161.5^{+21.4}_{-6.5}$
B_K (lattice value not in the fit)	$0.697^{+0.276}_{-0.051}$	$0.697^{+0.351}_{-0.074}$	$0.697^{+0.407}_{-0.097}$
f_{B_s}/f_{B_d} (lattice value not in the fit)	$1.242^{+0.043}_{-0.031}$	$1.242^{+0.076}_{-0.080}$	$1.24^{+0.11}_{-0.13}$
f_{B_s} (lattice value not in the fit)	$0.2259^{+0.0064}_{-0.0067}$	$0.2259^{+0.0130}_{-0.0090}$	$0.226^{+0.016}_{-0.011}$
B_{B_s}/B_{B_d} (lattice value not in the fit)	$1.128^{+0.052}_{-0.071}$	$1.13^{+0.11}_{-0.14}$	$1.13^{+0.16}_{-0.23}$
B_{B_s} (lattice value not in the fit)	$1.313^{+0.094}_{-0.042}$	$1.313^{+0.173}_{-0.065}$	$1.313^{+0.214}_{-0.088}$

(!) means that the quantity was not included in the fit.

Observable	**central** $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
$\mathcal{B}(B^+ \to \tau\nu)$ $[10^{-4}]$	$0.848\,^{+0.036}_{-0.055}$	$0.848\,^{+0.089}_{-0.084}$	$0.85\,^{+0.14}_{-0.11}$
$\mathcal{B}(B^+ \to \tau\nu)$ $[10^{-4}]$ (!)	$0.817\,^{+0.054}_{-0.031}$	$0.817\,^{+0.106}_{-0.060}$	$0.817\,^{+0.162}_{-0.088}$
$\mathcal{B}(B^+ \to \mu\nu)$ $[10^{-6}]$	$0.382\,^{+0.015}_{-0.025}$	$0.382\,^{+0.039}_{-0.038}$	$0.382\,^{+0.064}_{-0.051}$
$\mathcal{B}(B^+ \to e\nu)$ $[10^{-11}]$	$0.893\,^{+0.036}_{-0.059}$	$0.893\,^{+0.092}_{-0.089}$	$0.89\,^{+0.15}_{-0.12}$
$\mathcal{B}(B_d \to e^+e^-)$ $[10^{-15}]$	$2.26\,^{+0.17}_{-0.12}$	$2.26\,^{+0.22}_{-0.19}$	$2.26\,^{+0.27}_{-0.23}$
$\mathcal{B}(B_d \to \mu^+\mu^-)$ $[10^{-11}]$	$9.66\,^{+0.69}_{-0.54}$	$9.66\,^{+0.92}_{-0.83}$	$9.7\,^{+1.1}_{-1.0}$
$\mathcal{B}(B_s \to e^+e^-)$ $[10^{-14}]$	$7.48\,^{+0.55}_{-0.28}$	$7.48\,^{+0.66}_{-0.37}$	$7.48\,^{+0.76}_{-0.45}$
$\mathcal{B}(B_s \to \mu^+\mu^-)$ $[10^{-9}]$	$3.21\,^{+0.22}_{-0.14}$	$3.21\,^{+0.27}_{-0.17}$	$3.21\,^{+0.31}_{-0.21}$
$\mathcal{B}(B_s \to \mu^+\mu^-)$ $[10^{-9}]$ (!)	$3.31\,^{+0.14}_{-0.22}$	$3.31\,^{+0.18}_{-0.26}$	$3.31\,^{+0.22}_{-0.30}$
$\mathcal{B}(D_s \to \tau^+\nu)$ (!)	$0.05086\,^{+0.00112}_{-0.00065}$	$0.05086\,^{+0.00126}_{-0.00078}$	$0.05086\,^{+0.00139}_{-0.00091}$
$\mathcal{B}(D_s \to \mu^+\nu)$ $[10^{-2}]$ (!)	$0.5268\,^{+0.0068}_{-0.0114}$	$0.5268\,^{+0.0081}_{-0.0128}$	$0.5268\,^{+0.0094}_{-0.0141}$
$\mathcal{B}(D \to \mu^+\nu)$ $[10^{-3}]$ (!)	$0.4095\,^{+0.0036}_{-0.0078}$	$0.4095\,^{+0.0058}_{-0.0162}$	$0.4095\,^{+0.0077}_{-0.0204}$
$\mathcal{B}(K \to \mu^+\nu)$ (!)	$0.6357\,^{+0.0024}_{-0.0025}$	$0.6357\,^{+0.0046}_{-0.0054}$	$0.6357\,^{+0.0065}_{-0.0082}$
$\mathcal{B}(K \to e^+\nu)$ $[10^{-4}]$ (!)	$0.15689\,^{+0.00045}_{-0.00048}$	$0.15689\,^{+0.00091}_{-0.00093}$	$0.1569\,^{+0.0013}_{-0.0013}$
$\mathcal{B}(\tau^+ \to K\nu)$ $[10^{-2}]$ (!)	$0.7170\,^{+0.0015}_{-0.0015}$	$0.7170\,^{+0.0030}_{-0.0031}$	$0.7170\,^{+0.0045}_{-0.0046}$

(!) means that the quantity was not included in the fit.

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