

UPDATED RESULTS ON THE CKM MATRIX

Including results presented up to
FPCP 13

P r e l i m i n a r y

June 30th, 2013

The CKMfitter Group

Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach *Rfit*. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

CP VIOLATION AND THE CKM MATRIX:
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES

By CKMfitter Group

Eur. Phys. J. **C41**, 1-131, 2005 [hep-ph/0406184]

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1 Inputs

Parameter	Value \pm Error(s)	Reference	Errors	
			GS	TH
$ V_{ud} $ (nuclei)	0.97425 ± 0.00022	[1]	-	*
$ V_{us} $ ($K_{\ell 3}$)	$0.2258 \pm 0.0008 \pm 0.0012$	[4, 3]	*	*
$ V_{cd} $ (νN)	0.230 ± 0.011	[4]	*	-
$ V_{cs} $ ($W \rightarrow c\bar{s}$)	$0.94^{+0.32}_{-0.26} \pm 0.13$	[4]	*	*
$ V_{ub} $	$(3.75 \pm 0.14 \pm 0.26) \times 10^{-3}$	[5, 6]	*	*
$ V_{cb} $	$(41.15 \pm 0.33 \pm 0.59) \times 10^{-3}$	[5]	*	*
$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau)$	$(1.15 \pm 0.23) \times 10^{-4}$	[7]	*	-
$\mathcal{B}(D_s^- \rightarrow \mu^- \bar{\nu}_\mu)$	$(5.54 \pm 0.24) \times 10^{-3}$	[5]	*	-
$\mathcal{B}(D_s^- \rightarrow \tau^- \bar{\nu}_\tau)$	$(5.44 \pm 0.22) \times 10^{-2}$	[5]	*	-
$\mathcal{B}(D^- \rightarrow \mu^- \bar{\nu}_\mu)$	$(3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$	[8]	*	*
$\mathcal{B}(K^- \rightarrow e^- \bar{\nu}_e)$	$(1.581 \pm 0.008) \times 10^{-5}$	[4]	*	-
$\mathcal{B}(K^- \rightarrow \mu^- \bar{\nu}_\mu)$	0.6355 ± 0.0011	[2]	*	-
$\mathcal{B}(\tau^- \rightarrow K^- \bar{\nu}_\tau)$	$(0.700 \pm 0.010) \times 10^{-3}$	[4]	*	-
$\mathcal{B}(K^- \rightarrow \mu^- \bar{\nu}_\mu) / \mathcal{B}(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$	1.344 ± 0.0041	[2]	*	-
$\mathcal{B}(\tau^- \rightarrow K^- \bar{\nu}_\tau) / \mathcal{B}(\tau^- \rightarrow \pi^- \bar{\nu}_\tau)$	$(6.53 \pm 0.11) \times 10^{-2}$	[9]	*	-
$ V_{cd} f_+^{D \rightarrow \pi}(0)$	0.148 ± 0.004	[10]	*	-
$ V_{cs} f_+^{D \rightarrow K}(0)$	0.712 ± 0.007	[10, 11]	*	-
$ \varepsilon_K $	$(2.229 \pm 0.010) \times 10^{-3}$	[4]	*	-
Δm_d	$(0.507 \pm 0.004) \text{ ps}^{-1}$	[5]	*	-
Δm_s	$(17.768 \pm 0.024) \text{ ps}^{-1}$	[12]	*	-
$\sin(2\beta)_{[c\bar{c}]}$	0.679 ± 0.020	[5]	*	-
$(\beta_s)_{J/\psi\phi}$	Analysis of $B_s \rightarrow J/\psi\phi$	[13]	*	-
$S_{\pi\pi}^{+-}, C_{\pi\pi}^{+-}, C_{\pi\pi}^{00}, \mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis	[14]	*	-
$S_{\rho\rho,L}^{+-}, C_{\rho\rho,L}^{+-}, S_{\rho\rho}^{00}, C_{\rho\rho}^{00}, \mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[15]	*	-
$B_0^0 \rightarrow (\rho\pi)_0^0 \rightarrow 3\pi$	Time-dependent Dalitz analysis	[16]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to GLW analysis	[17]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to ADS analysis	[18]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	GGSZ Dalitz analysis	[19]	*	-

Table 1: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the ‘‘GS’’ column by a star; measurements or parameters that have systematic theoretical errors are marked in the ‘‘TH’’ column by a star. Upper part: experimental determinations of the CKM matrix elements. Lower part: CP-violation and mixing observables.*

Parameter	Value \pm Error(s)	Reference	Errors	
			GS	TH
$\overline{m}_c(m_c)$	$(1.286 \pm 0.013 \pm 0.040)$ GeV	[20]	*	*
$\overline{m}_t(m_t)$	$(165.8 \pm 0.54 \pm 0.72)$ GeV	[24]	*	*
$\alpha_s(m_Z)$	$0.1184 \pm 0 \pm 0.0007$	[4]	-	*
B_K	$0.7615 \pm 0.0026 \pm 0.0137$	[3]	*	*
κ_ϵ	$0.940 \pm 0.013 \pm 0.023$	[26, 27]	*	*
η_{cc}	$1.87 \pm 0 \pm 0.76$	[28]	-	*
η_{ct}	$0.497 \pm 0 \pm 0.047$	[29]	-	*
η_{tt}	$0.5765 \pm 0 \pm 0.0065$	[30]	-	*
$\eta_B(\overline{\text{MS}})$	$0.5510 \pm 0 \pm 0.0022$	[31, 32]	-	*
f_{B_s}	$(225.6 \pm 1.2 \pm 5.4)$ MeV	[3]	*	*
B_s	$1.326 \pm 0.016 \pm 0.040$	[3]	*	*
f_{B_s}/f_{B_d}	$1.198 \pm 0.008 \pm 0.025$	[3]	*	*
B_s/B_d	$1.036 \pm 0.013 \pm 0.023$	[3]	*	*
f_K	$(156.3 \pm 0.3 \pm 1.9)$ MeV	[3]	*	*
f_K/f_π	$1.1985 \pm 0.0013 \pm 0.0095$	[3]	*	*
f_{D_S}	$(248 \pm 1 \pm 5)$ MeV	[3]	*	*
f_{D_S}/f_D	$1.186 \pm 0.003 \pm 0.010$	[3]	*	*
$f_+^{D \rightarrow \pi}(0)$	$0.666 \pm 0.017 \pm 0.048$	[3]	*	*
$f_+^{D \rightarrow K}(0)$	$0.747 \pm 0.010 \pm 0.034$	[3]	*	*

Table 2: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the “GS” column by a star; measurements or parameters that have systematic theoretical errors are marked in the “TH” column by a star. Upper part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

2 Lattice QCD averages

Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

2.1 Method of averaging

We collect the relevant calculations of the quantity that we are interested in: we take only unquenched results with 2 or 2+1 dynamical fermions, even those from proceedings without a companion article. In these results, we separate the error estimates into a Gaussian part and a flat part that is treated à la Rfit. The Gaussian part collects the uncertainties from purely statistical origin, but also the systematics that can be controlled and treated in a similar way (e.g., interpolation or fitting in some cases). The remaining systematics constitute the Rfit error. If there are several sources of error in the Rfit category, we add them linearly ¹.

The Rfit model is simple but also very strict. It amounts to assuming that the theoretical uncertainty is rigorously constrained by a mathematical bound that is our only piece of information. If Rfit is taken *stricto sensu* and the individual likelihoods are combined in the usual way (by multiplication), the final uncertainty can be underestimated, in particular in the case of marginally compatible values.

We correct this effect by adopting the following averaging recipe. The central value is obtained by combining the whole likelihoods. Then we combine the Gaussian uncertainties by combining likelihoods restricted to their Gaussian part. Finally we assign to this combination the smallest of the individual Rfit uncertainties. The underlying idea is twofold:

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
- this best estimate should not be penalized by less precise methods (as it would happen be the case if one would take the dispersion of the individual central values as a guess of the combined theoretical uncertainty).

It should be stressed that the concept of a theoretical uncertainty is ill-defined, and the combination of them even more. Thus our approach is only one among the alternatives that can be found in the literature. In contrast to some of the latter, ours is algorithmic and can be reproduced.

2.2 Decay constants

2.2.1 Light mesons

f_K

¹keeping in mind that in many papers in the literature, this combination is done in quadrature and the splitting between different sources is not published.

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[40]	2	158.1	0.8	3.1
MILC07	[41]	2+1	156.5	0.4	+1.0 -2.7
HPQCD07	[42]	2+1	157	0.6	3.3
ALVdW08	[43]	2+1	153.9	1.7	6.5
Our average			156.3	0.3	1.9

f_K/f_π

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[40]	2	1.210	0.006	0.024
MILC07	[41]	2+1	1.197	0.003	+0.006 -0.013
NPLQCD07	[44]	2+1	1.218	0.002	+0.011 -0.024
HPQCD07	[42]	2+1	1.189	0.002	0.014
ALVdW08	[43]	2+1	1.191	0.016	0.026
BMW10	[45]	2+1	1.192	0.007	0.013
Our average			1.1985	0.0013	0.0095

2.2.2 Charmed mesons

f_{D_s}

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[40]	2	244	3	9
HPQCD10	[46]	2+1	248.0	1.4	4.5
HPQCD12	[47]	2+1	246.0	0.7	6.0
FNAL-MILC12	[49]	2+1	246.4	0.5	5.6
Our average			247.8	0.4	4.5

f_{D_s}/f_D

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[40]	2	1.24	0.03	0.01
HPQCD07	[42]	2+1	1.164	0.006	0.020
HPQCD12	[47]	2+1	1.187	0.004	0.023
FNAL-MILC12	[49]	2+1	1.175	0.016	0.018
Our average			1.186	0.003	0.010

2.2.3 Beauty mesons

f_{B_s}

Reference	Article	N_f	Mean	Stat	Syst
ETMC11	[50]	2	232	7	15
HPQCD11	[51]	2+1	225.0	2.9	5.4
FNAL-MILC11	[52]	2+1	242.0	5.1	21.2
HPQCD12	[53]	2+1	228.0	1.4	17.3
Our average			225.6	1.21	5.4

$$f_{B_s}/f_B$$

Reference	Article	N_f	Mean	Stat	Syst
ETMC11	[50]	2	1.19	0.02	0.06
RBC/UKQCD10	[54]	2+1	1.15	0.05	0.20
FNAL-MILC11	[52]	2+1	1.229	0.013	0.046
HPQCD12	[53]	2+1	1.188	0.012	0.025
Our average			1.198	0.008	0.025

2.3 Semileptonic form factors

2.3.1 $K \rightarrow \pi \ell \nu$

$$f_+(0)$$

Reference	Article	N_f	Mean	Stat	Syst
JLQCD12	[55]	2	0.959	0.006	0.007
ETMC09	[56]	2	0.9560	0.0057	0.0118
RBC-UKQCD10	[57]	2+1	0.9599	0.0034	$+0.0045$ -0.0057
Our average			0.9593	0.0026	0.0051

Combining with $|V_{us}|f_+(0) = 0.2166(5)$ from ref.[58], we get $|V_{us}| = 0.2258 \pm 0.0008 \pm 0.0012$.

2.3.2 $D \rightarrow \pi \ell \nu$

$$f_+(0)$$

Reference	Article	N_f	Mean	Stat	Syst
ETMC11	[59]	2	0.65	0.06	0.12
MILC04	[60]	2+1	0.64	0.03	0.15
HPQCD11	[61]	2+1	0.666	0.021	0.048
Our average			0.666	0.017	0.048

2.3.3 $D \rightarrow K \ell \nu$

$$f_+(0)$$

Reference	Article	N_f	Mean	Stat	Syst
ETMC11	[59]	2	0.76	0.05	0.11
MILC04	[60]	2+1	0.73	0.03	0.15
HPQCD10	[62]	2+1	0.747	0.011	0.034
Our average			0.747	0.010	0.034

2.4 Meson mixing

2.4.1 Kaon mixing

$B_K^{\text{MS}}(2\text{GeV})$

Reference	Article	N_f	Mean	Stat	Syst
JLQCD08	[63]	2	0.537	0.004	0.072
ETMC 10	[64]	2	0.532	0.019	0.026
HPQCD/UKQCD06	[65]	2+1	0.618	0.018	0.179
ALVdW09	[66]	2+1	0.527	0.006	0.035
RBC/UKQCD10	[67]	2+1	0.549	0.005	0.038
SWME11	[68]	2+1	0.530	0.003	0.051
BMW11	[69]	2+1	0.5644	0.0059	0.0100
Our average for $B_K^{\text{MS}}(2\text{GeV})$			0.5562	0.0019	0.0100
Our average for \hat{B}_K			0.7615	0.0026	0.0137

2.4.2 $B_{d,s}$ mixing

\hat{B}_{B_s}

Reference	Article	N_f	Mean	Stat	Syst
JLQCD03	[70]	2	1.299	0.034	$+0.121$ -0.087
HPQCD09	[71]	2+1	1.326	0.018	0.040
Our average			1.326	0.016	0.040

Ref. [71] provide only f_{B_s} and $f_{B_s}\sqrt{\hat{B}_{B_s}}$, and we assumed that the systematics were completely correlated to extract \hat{B}_{B_s} .

$\hat{B}_{B_s}/\hat{B}_{B_d}$

Reference	Article	N_f	Mean	Stat	Syst
JLQCD03	[70]	2	1.017	0.016	$+0.076$ -0.017
HPQCD09	[71]	2+1	1.053	0.025	0.023
RBC/UKQCD10	[54]	2+1	0.966	0.038	0.077
FNAL/MILC12	[48]	2+1	1.064	0.076	0.193
Our average			1.036	0.013	0.023

Refs. [71], [54] and [48] provide only ξ and f_{B_s}/f_{B_d} . For Refs. [71] and [54], we have extracted $\hat{B}_{B_s}/\hat{B}_{B_d}$ in both cases assuming a total correlation in the systematics of ξ and $\hat{B}_{B_s}/\hat{B}_{B_d}$. For Ref. [48], we have considered all uncertainties as uncorrelated, as the studies of the decay constants and the bag parameters have been performed in different settings, with different categories of systematics.

3 Results

Observable	central \pm CL \equiv 1 σ	\pm CL \equiv 2 σ	\pm CL \equiv 3 σ
A	$0.823^{+0.012}_{-0.033}$	$0.823^{+0.018}_{-0.042}$	$0.823^{+0.025}_{-0.049}$
λ	$0.22457^{+0.00186}_{-0.00014}$	$0.22457^{+0.00193}_{-0.00021}$	$0.22457^{+0.00200}_{-0.00027}$
$\bar{\rho}$	$0.1289^{+0.0176}_{-0.0094}$	$0.129^{+0.056}_{-0.018}$	$0.129^{+0.075}_{-0.027}$
$\bar{\eta}$	$0.348^{+0.012}_{-0.012}$	$0.348^{+0.025}_{-0.030}$	$0.348^{+0.037}_{-0.044}$
J [10^{-5}]	$3.02^{+0.16}_{-0.19}$	$3.02^{+0.29}_{-0.30}$	$3.02^{+0.42}_{-0.36}$
$\sin 2\alpha$	$0.052^{+0.052}_{-0.097}$	$0.05^{+0.10}_{-0.32}$	$0.05^{+0.15}_{-0.43}$
$\sin 2\alpha$ (!)	$-0.17^{+0.24}_{-0.16}$	$-0.17^{+0.30}_{-0.22}$	$-0.17^{+0.36}_{-0.27}$
$\sin 2\beta$	$0.689^{+0.019}_{-0.019}$	$0.689^{+0.039}_{-0.035}$	$0.689^{+0.058}_{-0.050}$
$\sin 2\beta$ (!)	$0.778^{+0.020}_{-0.048}$	$0.778^{+0.039}_{-0.108}$	$0.778^{+0.058}_{-0.143}$
α [$^\circ$]	$88.5^{+2.8}_{-1.5}$	$88.5^{+9.2}_{-2.9}$	$88.5^{+12.5}_{-4.3}$
α [$^\circ$] (!)	$94.9^{+4.8}_{-6.9}$	$94.9^{+6.6}_{-8.8}$	$94.9^{+8.2}_{-10.4}$
α [$^\circ$] (dir. meas.)	$85.4^{+4.0}_{-3.9}$	$85.4^{+10.4}_{-7.8} 0.2^{+9.6}_{-8.8}$	$85^{+17}_{-12} 0^{+15}_{-15}$
β [$^\circ$]	$21.79^{+0.78}_{-0.73}$	$21.8^{+1.6}_{-1.3}$	$21.8^{+2.4}_{-1.9}$
β [$^\circ$] (!)	$25.53^{+0.92}_{-2.07}$	$25.5^{+1.9}_{-4.5}$	$25.5^{+2.8}_{-5.8}$
β [$^\circ$] (dir. meas.)	$21.38^{+0.79}_{-0.77}$	$21.4^{+1.6}_{-1.5}$	$21.4^{+2.4}_{-2.3}$
γ [$^\circ$]	$69.7^{+1.3}_{-2.8}$	$69.7^{+2.6}_{-9.1}$	$69.7^{+3.9}_{-11.9}$
γ [$^\circ$] (!)	$69.7^{+1.3}_{-2.8}$	$69.7^{+2.7}_{-10.2}$	$69.7^{+4.0}_{-12.4}$
γ [$^\circ$] (dir. meas.)	$68.0^{+8.0}_{-8.5}$	68^{+15}_{-17}	68^{+23}_{-25}
R_u	$0.371^{+0.013}_{-0.012}$	$0.371^{+0.026}_{-0.022}$	$0.371^{+0.039}_{-0.031}$
R_t	$0.9381^{+0.0087}_{-0.0179}$	$0.938^{+0.017}_{-0.059}$	$0.938^{+0.025}_{-0.078}$
$\bar{\rho}_s$	$-0.00684^{+0.00051}_{-0.00104}$	$-0.00684^{+0.00098}_{-0.00314}$	$-0.0068^{+0.0014}_{-0.0042}$
$\bar{\eta}_s$	$-0.01851^{+0.00064}_{-0.00073}$	$-0.0185^{+0.0015}_{-0.0015}$	$-0.0185^{+0.0023}_{-0.0021}$
$\beta_s \equiv -\arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right)$ [rad]	$0.01838^{+0.00072}_{-0.00064}$	$0.0184^{+0.0014}_{-0.0016}$	$0.0184^{+0.0021}_{-0.0023}$
$\sin 2\beta_s$	$0.0367^{+0.0014}_{-0.0013}$	$0.0367^{+0.0029}_{-0.0031}$	$0.0367^{+0.0042}_{-0.0046}$

Observable	central \pm CL \equiv 1 σ	\pm CL \equiv 2 σ	\pm CL \equiv 3 σ
$ V_{ud} $	0.974452 ^{+0.000033} _{-0.000432}	0.974452 ^{+0.000048} _{-0.000449}	0.974452 ^{+0.000063} _{-0.000465}
$ V_{us} $	0.22457 ^{+0.00186} _{-0.00014}	0.22457 ^{+0.00194} _{-0.00021}	0.22457 ^{+0.00200} _{-0.00028}
$ V_{ub} $	0.00355 ^{+0.00016} _{-0.00013}	0.00355 ^{+0.00030} _{-0.00024}	0.00355 ^{+0.00043} _{-0.00033}
$ V_{cd} $	0.22443 ^{+0.00186} _{-0.00015}	0.22443 ^{+0.00194} _{-0.00021}	0.22443 ^{+0.00200} _{-0.00028}
$ V_{cs} $	0.973607 ^{+0.000069} _{-0.000445}	0.973607 ^{+0.000092} _{-0.000468}	0.97361 ^{+0.00011} _{-0.00049}
$ V_{cb} $	0.04151 ^{+0.00056} _{-0.00115}	0.04151 ^{+0.00089} _{-0.00153}	0.0415 ^{+0.0012} _{-0.0019}
$ V_{td} $	0.00875 ^{+0.00016} _{-0.00031}	0.00875 ^{+0.00026} _{-0.00065}	0.00875 ^{+0.00037} _{-0.00081}
$ V_{ts} $	0.04073 ^{+0.00055} _{-0.00112}	0.04073 ^{+0.00088} _{-0.00149}	0.0407 ^{+0.0012} _{-0.0018}
$ V_{tb} $	0.999132 ^{+0.000047} _{-0.000024}	0.999132 ^{+0.000063} _{-0.000038}	0.999132 ^{+0.000076} _{-0.000052}
$ V_{ud} $ (!)	0.97446 ^{+0.00018} _{-0.00065}	0.97446 ^{+0.00036} _{-0.00082}	0.97446 ^{+0.00054} _{-0.00093}
$ V_{us} $ (!)	0.224489 ^{+0.001948} _{-0.000066}	0.22449 ^{+0.00202} _{-0.00013}	0.22449 ^{+0.00209} _{-0.00020}
$ V_{ub} $ (!)	0.00343 ^{+0.00023} _{-0.00012}	0.00343 ^{+0.00037} _{-0.00023}	0.00343 ^{+0.00051} _{-0.00031}
$ V_{cb} $ (!)	0.0415 ^{+0.0024} _{-0.0016}	0.0415 ^{+0.0028} _{-0.0021}	0.0415 ^{+0.0031} _{-0.0024}
Δm_d [ps ⁻¹] (!)	0.596 ^{+0.047} _{-0.076}	0.596 ^{+0.081} _{-0.125}	0.60 ^{+0.12} _{-0.17}
Δm_s [ps ⁻¹] (!)	16.5 ^{+1.8} _{-1.4}	16.5 ^{+3.3} _{-1.8}	16.5 ^{+4.0} _{-2.1}
$ \epsilon_K $ [10 ⁻³] (!)	2.16 ^{+0.56} _{-0.56}	2.16 ^{+0.69} _{-0.71}	2.16 ^{+0.82} _{-0.81}
m_t [GeV/c ²] (!)	160.1 ^{+13.7} _{-5.9}	160.1 ^{+24.8} _{-8.0}	160 ⁺²⁸ ₋₁₀
B_K (lattice value not in the fit)	0.81 ^{+0.24} _{-0.17}	0.81 ^{+0.34} _{-0.20}	0.81 ^{+0.43} _{-0.23}
f_{B_s}/f_{B_d} (lattice value not in the fit)	1.240 ^{+0.058} _{-0.038}	1.240 ^{+0.098} _{-0.099}	1.24 ^{+0.14} _{-0.15}
f_{B_s} (lattice value not in the fit)	0.2350 ^{+0.0038} _{-0.0152}	0.2350 ^{+0.0063} _{-0.0184}	0.2350 ^{+0.0089} _{-0.0208}
B_{B_s}/B_{B_d} (lattice value not in the fit)	1.218 ^{+0.065} _{-0.130}	1.22 ^{+0.13} _{-0.23}	1.22 ^{+0.20} _{-0.33}
B_{B_s} (lattice value not in the fit)	1.255 ^{+0.135} _{-0.038}	1.255 ^{+0.267} _{-0.061}	1.255 ^{+0.304} _{-0.084}

Observable	central \pm CL \equiv 1 σ	\pm CL \equiv 2 σ	\pm CL \equiv 3 σ
$\mathcal{B}(B^+ \rightarrow \tau\nu)$ [10 ⁻⁴]	0.794 ^{+0.062} _{-0.077}	0.79 ^{+0.16} _{-0.14}	0.79 ^{+0.25} _{-0.18}
$\mathcal{B}(B^+ \rightarrow \tau\nu)$ [10 ⁻⁴] (!)	0.739 ^{+0.091} _{-0.071}	0.74 ^{+0.17} _{-0.11}	0.74 ^{+0.27} _{-0.14}
$\mathcal{B}(B^+ \rightarrow \mu\nu)$ [10 ⁻⁶]	0.357 ^{+0.028} _{-0.035}	0.357 ^{+0.073} _{-0.063}	0.357 ^{+0.111} _{-0.081}
$\mathcal{B}(B^+ \rightarrow e\nu)$ [10 ⁻¹¹]	0.835 ^{+0.066} _{-0.081}	0.84 ^{+0.17} _{-0.15}	0.84 ^{+0.26} _{-0.19}
$\mathcal{B}(B_d \rightarrow e^+e^-)$ [10 ⁻¹⁵]	2.60 ^{+0.15} _{-0.26}	2.60 ^{+0.21} _{-0.39}	2.60 ^{+0.26} _{-0.44}
$\mathcal{B}(B_d \rightarrow \mu^+\mu^-)$ [10 ⁻¹¹]	11.10 ^{+0.64} _{-1.10}	11.10 ^{+0.88} _{-1.66}	11.1 ^{+1.1} _{-1.9}
$\mathcal{B}(B_s \rightarrow e^+e^-)$ [10 ⁻¹⁴]	8.31 ^{+0.43} _{-0.79}	8.31 ^{+0.55} _{-0.91}	8.31 ^{+0.68} _{-1.01}
$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ [10 ⁻⁹]	3.55 ^{+0.18} _{-0.34}	3.55 ^{+0.24} _{-0.39}	3.55 ^{+0.29} _{-0.43}
$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ [10 ⁻⁹] (!)	3.55 ^{+0.18} _{-0.34}	3.55 ^{+0.24} _{-0.39}	3.55 ^{+0.29} _{-0.43}
$\mathcal{B}(D_s \rightarrow \tau^+\nu)$ (!)	0.05290 ^{+0.00017} _{-0.00150}	0.05290 ^{+0.00034} _{-0.00369}	0.05290 ^{+0.00051} _{-0.00408}
$\mathcal{B}(D_s \rightarrow \mu^+\nu)$ [10 ⁻²] (!)	0.5422 ^{+0.0017} _{-0.0124}	0.5422 ^{+0.0035} _{-0.0326}	0.5422 ^{+0.0052} _{-0.0414}
$\mathcal{B}(D \rightarrow \mu^+\nu)$ [10 ⁻³] (!)	0.405 ^{+0.016} _{-0.013}	0.405 ^{+0.019} _{-0.024}	0.405 ^{+0.022} _{-0.035}
$\mathcal{B}(K \rightarrow \mu^+\nu)$ (!)	0.6376 ^{+0.0031} _{-0.0031}	0.6376 ^{+0.0061} _{-0.0061}	0.6376 ^{+0.0092} _{-0.0092}
$\mathcal{B}(K \rightarrow e^+\nu)$ [10 ⁻⁴] (!)	0.15693 ^{+0.00047} _{-0.00047}	0.15693 ^{+0.00093} _{-0.00093}	0.1569 ^{+0.0013} _{-0.0013}
$\mathcal{B}(\tau^+ \rightarrow K\nu)$ [10 ⁻²] (!)	0.7179 ^{+0.0015} _{-0.0015}	0.7179 ^{+0.0031} _{-0.0031}	0.7179 ^{+0.0046} _{-0.0046}

4 New Physics in $\Delta F = 2$

We follow refs. [27, 33], and consider Scenario I where New Physics enters B_d and B_s mixings through $M_{12}^q = M_{12;SM}^q \Delta_q$. The two NP complex parameters Δ_q are uncorrelated (Scenario I). In addition to the inputs presented above and below, we use the additional theoretical inputs discussed in refs. [27, 33] necessary to describe NP in $\Delta F = 2$

4.1 Inputs

Parameter	Value \pm Error(s)	Reference	Errors	
			GS	TH
a_{SL}^d	0.0023 ± 0.0026	[5, 34, 35]	*	-
a_{SL}^s	-0.0022 ± 0.0052	[36, 37]	*	-
A_{SL}	-0.0074 ± 0.0019	[38, 39]	*	-
ϕ_s	$-0.013^{+0.083}_{-0.090}$	[5]	*	-
$\Delta\Gamma_s$	0.091 ± 0.011	[5]	*	-

4.2 Results

Observable	central \pm CL \equiv 1σ	\pm CL \equiv 2σ	\pm CL \equiv 3σ
Re(Δ_d)	0.803 ^{+0.269} _{-0.097}	0.80 ^{+0.42} _{-0.14}	0.80 ^{+0.57} _{-0.17}
Im(Δ_d)	-0.111 ^{+0.073} _{-0.060}	-0.11 ^{+0.14} _{-0.13}	-0.11 ^{+0.19} _{-0.19}
$ \Delta_d $	0.81 ^{+0.27} _{-0.10}	0.81 ^{+0.42} _{-0.14}	0.81 ^{+0.58} _{-0.18}
ϕ_d^Δ [deg]	-7.9 ^{+5.0} _{-2.3}	-7.9 ^{+10.1} _{-4.6}	-7.9 ^{+13.3} _{-7.0}
Re(Δ_s)	0.970 ^{+0.201} _{-0.079}	0.970 ^{+0.234} _{-0.099}	0.97 ^{+0.26} _{-0.12}
Im(Δ_s)	-0.005 ^{+0.089} _{-0.092}	-0.00 ^{+0.18} _{-0.19}	-0.00 ^{+0.28} _{-0.30}
$ \Delta_s $	0.973 ^{+0.198} _{-0.082}	0.97 ^{+0.23} _{-0.10}	0.97 ^{+0.26} _{-0.12}
ϕ_s^Δ [deg]	-0.3 ^{+5.1} _{-5.3}	-0. ^{+10.} _{-11.}	-0. ^{+15.} _{-16.}
$\phi_d^\Delta + 2\beta$ [deg] (!)	42. ^{+12.} _{-11.}	42. ^{+29.} _{-25.}	42. ^{+47.} _{-42.}
$\phi_s^\Delta - 2\beta_s$ [deg] (!)	-60.3 ^{+9.0} _{-4.5}	-60.3 ^{+29.7} _{-8.9}	-60. ^{+64.} _{-13.}
A_{SL} (!)	-0.00064 ^{+0.00046} _{-0.00054}	-0.00064 ^{+0.00093} _{-0.00106}	-0.0006 ^{+0.0013} _{-0.0015}
	-0.00064 ^{+0.00046} _{-0.00054}	or 0.00400 ^{+0.00055} _{-0.00035}	or 0.0040 ^{+0.0020} _{-0.0011}
A_{SL}	-0.00127 ^{+0.00064} _{-0.00033}	-0.00127 ^{+0.00113} _{-0.00067}	-0.0013 ^{+0.0016} _{-0.0010}
$a_{\text{SL}}^s - a_{\text{SL}}^d$	0.00234 ^{+0.00066} _{-0.00126}	0.0023 ^{+0.0013} _{-0.0023}	0.0023 ^{+0.0020} _{-0.0032}
a_{SL}^d (!)	-0.00246 ^{+0.00063} _{-0.00045}	-0.00246 ^{+0.00163} _{-0.00089}	-0.0025 ^{+0.0024} _{-0.0014}
a_{SL}^s (!)	0.00003 ^{+0.00046} _{-0.00048}	0.00003 ^{+0.00092} _{-0.00099}	0.0000 ^{+0.0014} _{-0.0015}
$\Delta\Gamma_d$ [ps ⁻¹]	0.00476 ^{+0.00073} _{-0.00231}	0.0048 ^{+0.0013} _{-0.0028}	0.0048 ^{+0.0018} _{-0.0031}
$\Delta\Gamma_s$ [ps ⁻¹] (!)	0.155 ^{+0.020} _{-0.086}	0.155 ^{+0.026} _{-0.092}	0.155 ^{+0.032} _{-0.098}
$\Delta\Gamma_s$ [ps ⁻¹]	0.091 ^{+0.011} _{-0.011}	0.091 ^{+0.022} _{-0.021}	0.091 ^{+0.033} _{-0.028}
$B \rightarrow \tau\nu$ [10 ⁻⁴] (!)	1.109 ^{+0.081} _{-0.394}	1.11 ^{+0.16} _{-0.50}	1.11 ^{+0.25} _{-0.56}
$B \rightarrow \tau\nu$ [10 ⁻⁴]	1.113 ^{+0.076} _{-0.176}	1.11 ^{+0.16} _{-0.37}	1.11 ^{+0.24} _{-0.50}

References

- [1] I.S. Towner and J.C. Hardy, Phys. Rev. **C 79**, 055502 (2009), arXiv:0812.1202 [nucl-ex].
- [2] FlaviaNet Working Group on Kaon Decays, Eur. Phys. J. **C69** (2010) 399-424. [arXiv:1005.2323 [hep-ph]].
- [3] See sec. 2.
- [4] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86** (2012) 010001.
- [5] The Heavy Flavor Averaging Group (HFAG), Fall 2012, <http://www.slac.stanford.edu/xorg/hfag>, and references therein. For charm semileptonic and leptonic decays, on "Charm 10" results.
- [6] For the inclusive average we are taking the BLNP number. (The DGE result is very close to the BLNP result. The uncertainties between BLNP and DGE are hard to compare.) The theoretical error on the inclusive average is obtained by adding linearly the contributions from weak annihilation, subleading shape functions and the HQE uncertainty on m_b . We use only branching fractions measured for $B \rightarrow \pi \ell \nu$. We average the results obtained from the two unquenched Lattice calculations quoted by HFAG [5], adding the systematic uncertainties in such a way that the smallest theoretical error is kept. We keep the smallest theoretical error also for the average between the inclusive and exclusive results.
- [7] J. P. Lees *et al.*, arXiv:1207.0698 [hep-ex],
B. Aubert *et al.*, Phys. Rev. **D 81**, 051101(R) (2010),
I. Adachi *et al.*, arXiv:1208.4678 [hep-ex],
K. Hara *et al.*, Phys. Rev. **D 82** (2010) 071101 [arXiv:1006.4201 [hep-ex]].
- [8] B. I. Eisenstein *et al.* [CLEO Collaboration], Phys. Rev. **D78** (2008) 052003. [arXiv:0806.2112 [hep-ex]].
- [9] S. Banerjee [BaBar Collaboration], [arXiv:0811.1429 [hep-ex]].
- [10] D. Besson *et al.* [CLEO Collaboration], Phys. Rev. D **80** (2009) 032005 [arXiv:0906.2983 [hep-ex]].
L. Widhalm *et al.* [Belle Collaboration], Phys. Rev. Lett. **97** (2006) 061804 [hep-ex/0604049].
- [11] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **76** (2007) 052005 [arXiv:0704.0020 [hep-ex]].
- [12] RAaij *et al.* [LHCb Collaboration], New J. Phys. **15** (2013) 053021 [arXiv:1304.4741 [hep-ex]].
- [13] The input on β_s is obtained by combining the available 2D $(\Delta\Gamma_s, \beta_s)$ likelihoods [5] obtained from the analysis of $B_s \rightarrow J/\psi\phi$. The information on $\Delta\Gamma_s$ is not exploited for the Standard Model global fit.
- [14] BaBar Collaboration, Phys. Rev. D75, 012008 (2007).
BaBar Collaboration, Phys.Rev. D87 (2013) 052009.
BaBar Collaboration, Phys. Rev. D76, 091102 (2007).
Belle Collaboration, Phys. Rev. D87, 031103 (2013).

- Belle collaboration, arXiv:1302.0551.
- Belle Collaboration, arXiv:hep-ex/0610065.
- CDF Collaboration, Phys. Rev. Lett. 106, 181802 (2011).
- Cleo Collaboration, Phys. Rev. D68, 052002 (2003).
- [15] BaBar Collaboration, Phys. Rev. D76, 052007 (2007).
 BaBar Collaboration, Phys. Rev. Lett. 102, 141802 (2009).
 BaBar Collaboration, Phys. Rev. D78, 071104 (2008).
 Belle Collaboration, Phys. Rev. Lett. 96, 171801 (2006).
 Belle Collaboration, Phys. Rev. D 76, 011104 (2007).
 Belle Collaboration, Phys. Rev. Lett. 91, 221801 (2003).
 Belle Collaboration, arXiv:hep-ex/1212.4015v1.
- [16] Babar Collaboration, Phys. Rev. D 88, 012003 (2013).
 Belle Collaboration, Phys. Rev. Lett. 98, 221602 (2007).
- [17] P. del Amo Sanchez *et al.*[BaBar Collaboration], Phys. Rev. D **82** (2010) 072004.
 B. Aubert *et al* [BaBar Collaboration] Phys. Rev. D **78** (2008) 092002.
 B. Aubert *et al* [BaBar Collaboration], Phys. Rev. D **80** (2009) 092001.
 K. Trabelsi for the Belle Collaboration, arXiv:1301.2033, preliminary results.
 R. Aaij *et al* [LHCb Collaboration] Phys.Lett. **B712** (2012) 203.
- [18] P. del Amo Sanchez *et al* [BaBar Collaboration], Phys.Rev. **D82** (2010) 072006.
 Y. Horii *et al* [Belle Collaboration], Phys.Rev.Lett. **106** (2011) 231803.
 K. Trabelsi for the Belle Collaboration, arXiv:1301.2033, preliminary results.
 R. Aaij *et al* [LHCb Collaboration], Phys.Lett. **B712** (2012) 203.
- [19] P. del Amo Sanchez *et al* [BaBar Collaboration], Phys.Rev.Lett. 105 (2010) 121801.
 A. Poluektov *et al* [Belle Collaboration], Phys.Rev. D81 (2010) 112002.
 LHCb Collaboration, LHCb-CONF-2013-004 [CERN-LHCb-CONF-2013-004], preliminary results.
- [20] We take $\bar{m}_c(m_c) = (1.286 \pm 0.013)$ GeV from an analysis by Kuhn and Steinhauser [23]. We assign an additional theoretical uncertainty of 0.040 GeV in order to take into account: a) an observed difference in analyses with participation of the same authors for the central value when using a somehow different extraction method [21, 22], and b) in order to take into account a likely over-optimistic error range for the gluon condensate in this analysis. This input is consistent with $\bar{m}_c(m_c) = (1.265 \pm 0.060 \pm 0.050)$ GeV translated from the kinetic charm mass obtained from fits to data from lepton energy and hadronic mass moments in $B \rightarrow X_c \ell \nu$ decays combined with photon energy moments measured in $B \rightarrow X_s \gamma$ decays: $m_c^{kin} = (1.165 \pm 0.050)$ GeV [5].
- [21] I. Allison *et al.* [HPQCD Collaboration], Phys. Rev. D **78** (2008) 054513 [arXiv:0805.2999 [hep-lat]].

- [22] M. Steinhauser, arXiv:0809.1925 [hep-ph].
- [23] J. H. Kuhn, M. Steinhauser and C. Sturm, Nucl. Phys. B **778** (2007) 192 [arXiv:hep-ph/0702103].
- [24] The world average of the top quark mass measurements performed at the Tevatron [25] interpreted as a pole mass is translated into $m_t(m_t)$ in the $\overline{\text{MS}}$ at 1-loop order.
- [25] [Tevatron Electroweak Working Group and for the CDF and D0 Collaborations], arXiv:1107.5255 [hep-ex].
- [26] A. J. Buras and D. Guadagnoli, Phys. Rev. D **78** (2008) 033005 [arXiv:0805.3887 [hep-ph]].
A. J. Buras, D. Guadagnoli and G. Isidori, Phys. Lett. B **688** (2010) 309 [arXiv:1002.3612 [hep-ph]].
- [27] A. Lenz, U. Nierste, J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker and S. Monteil *et al.*, Phys. Rev. D **83** (2011) 036004 [arXiv:1008.1593 [hep-ph]].
- [28] J. Brod and M. Gorbahn, Phys. Rev. Lett. **108** (2012) 121801 [arXiv:1108.2036 [hep-ph]].
- [29] J. Brod and M. Gorbahn, Phys. Rev. D **82** (2010) 094026 [arXiv:1007.0684 [hep-ph]].
- [30] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347** (1990) 491.
S. Herrlich and U. Nierste, Nucl. Phys. **B 419**, 292 (1994).
- [31] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
- [32] A. Lenz, private communication (2010).
- [33] A. Lenz, U. Nierste, J. Charles, S. Descotes-Genon, H. Lacker, S. Monteil, V. Niess and S. T'Jampens, Phys. Rev. D **86** (2012) 033008 [arXiv:1203.0238 [hep-ph]].
- [34] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **86** (2012) 072009 [arXiv:1208.5813 [hep-ex]].
- [35] J. P. Lees *et al.* [BaBar Collaboration], arXiv:1305.1575 [hep-ex].
- [36] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **82** (2010) 012003 [Erratum-ibid. D **83** (2011) 119901] [arXiv:0904.3907 [hep-ex]].
- [37] The LHCb Collaboration, LHCb-CONF-2012-022.
- [38] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **84** (2011) 052007 [arXiv:1106.6308 [hep-ex]].
- [39] The CDF Collaboration, <http://www-cdf.fnal.gov/physics/new/bottom/070816.blessed-acp-bsemil/>
- [40] B. Blossier *et al.*, JHEP **0907** (2009) 043 [arXiv:0904.0954 [hep-lat]].
- [41] C. Bernard *et al.*, PoS **LAT2007**, 090 (2007) [arXiv:0710.1118 [hep-lat]].
- [42] E. Follana, C. T. H. Davies, G. P. Lepage and J. Shigemitsu [HPQCD Collaboration and UKQCD Collaboration], Phys. Rev. Lett. **100**, 062002 (2008) [arXiv:0706.1726 [hep-lat]].

- [43] C. Aubin, J. Laiho and R. S. Van de Water, PoS LATTICE **2008** (2008) 105 [arXiv:0810.4328 [hep-lat]].
- [44] S. R. Beane, P. F. Bedaque, K. Orginos and M. J. Savage, Phys. Rev. D **75**, 094501 (2007) [arXiv:hep-lat/0606023].
- [45] S. Durr *et al.*, Phys. Rev. D **81** (2010) 054507 [arXiv:1001.4692 [hep-lat]].
- [46] C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na and J. Shigemitsu, Phys. Rev. D **82** (2010) 114504 [arXiv:1008.4018 [hep-lat]].
- [47] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **86** (2012) 054510 [arXiv:1206.4936 [hep-lat]].
- [48] A. Bazavov, C. Bernard, C. M. Bouchard, C. DeTar, M. Di Pierro, A. X. El-Khadra, R. T. Evans and E. D. Freeland *et al.*, Phys. Rev. D **86** (2012) 034503 [arXiv:1205.7013 [hep-lat]].
- [49] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaborations], PoS LATTICE **2012** (2012) 159 [arXiv:1210.8431 [hep-lat]].
- [50] P. Dimopoulos *et al.* [ETM Collaboration], JHEP **1201** (2012) 046 [arXiv:1107.1441 [hep-lat]].
- [51] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel and G. P. Lepage, Phys. Rev. D **85** (2012) 031503 [arXiv:1110.4510 [hep-lat]].
- [52] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaboration], Phys. Rev. D **85** (2012) 114506 [arXiv:1112.3051 [hep-lat]].
- [53] H. Na, C. J. Monahan, C. T. H. Davies, R. Horgan, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **86** (2012) 034506 [arXiv:1202.4914 [hep-lat]].
- [54] C. Albertus *et al.*, Phys. Rev. D **82** (2010) 014505 [arXiv:1001.2023 [hep-lat]].
- [55] T. Kaneko *et al.* [JLQCD Collaboration], PoS LATTICE **2012** (2012) 111 [arXiv:1211.6180 [hep-lat]].
- [56] V. Lubicz, F. Mescia, S. Simula, C. Tarantino and f. t. E. Collaboration, Phys. Rev. D **80** (2009) 111502 [arXiv:0906.4728 [hep-lat]].
- [57] P. A. Boyle *et al.*, Eur. Phys. J. C **69** (2010) 159 [arXiv:1004.0886 [hep-lat]].
- [58] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86** (2012) 010001.
- [59] S. Di Vita, B. Haas, V. Lubicz, F. Mescia, S. Simula *et al.*, PoS **LAT2010** (2010) . [arXiv:1104.0869 [hep-lat]].
- [60] C. Aubin *et al.* [Fermilab Lattice Collaboration and MILC Collaboration and HPQCD Collab], Phys. Rev. Lett. **94** (2005) 011601 [arXiv:hep-ph/0408306].
- [61] H. Na, C. T. H. Davies, E. Follana, J. Koponen, G. P. Lepage and J. Shigemitsu, Phys-RevD.84.114505,2011 [arXiv:1109.1501 [hep-lat]].

- [62] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **82**, 114506 (2010) [arXiv:1008.4562 [hep-lat]].
- [63] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. D **77** (2008) 094503 [arXiv:0801.4186 [hep-lat]].
- [64] M. Constantinou *et al.* [ETM Collaboration], Phys. Rev. D **83** (2011) 014505 [arXiv:1009.5606 [hep-lat]].
- [65] E. Gamiz, S. Collins, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration and UKQCD Collaboration], Phys. Rev. D **73**, 114502 (2006) [arXiv:hep-lat/0603023].
- [66] C. Aubin, J. Laiho and R. S. Van de Water, Phys. Rev. D **81** (2010) 014507 [arXiv:0905.3947 [hep-lat]].
- [67] Y. Aoki, R. Arthur, T. Blum, P. A. Boyle, D. Brommel, N. H. Christ, C. Dawson and T. Izubuchi *et al.*, Phys. Rev. D **84** (2011) 014503 [arXiv:1012.4178 [hep-lat]].
- [68] W. Lee, Y. -C. Jang, H. -J. Kim, J. Kim, K. Kim, B. Yoon, T. Bae and C. Jung *et al.*, PoS LATTICE **2011** (2011) 316 [arXiv:1110.2576 [hep-lat]].
- [69] S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch and T. Lippert *et al.*, Phys. Lett. B **705** (2011) 477 [arXiv:1106.3230 [hep-lat]].
- [70] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. Lett. **91**, 212001 (2003) [arXiv:hep-ph/0307039].
- [71] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Phys. Rev. D **80**, 014503 (2009) [arXiv:0902.1815 [hep-lat]].