Average of lattice QCD inputs for CKM fits

April 26, 2010

The CKMfitter group

Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

1 Method of averaging

We collect the relevant calculations of the quantity that we are interested in: we take only unquenched results with 2 or 2+1 dynamical fermions, even those from proceedings without a companion article (flagged with a star). In these results, we separate the error estimates into a Gaussian part and a flat part that is treated à la Rfit. The Gaussian part collects the uncertainties from purely statistical origin, but also the systematics that can be controlled and treated in a similar way (e.g., interpolation or fitting in some cases). The remaining systematics constitute the Rfit error. If there are several sources of error in the Rfit category, we add them linearly [1].

The Rfit model is simple but also very strict. It amounts to assuming that the theoretical uncertainty is rigorously constrained by a mathematical bound that is our only piece of information. If Rfit is taken stricto sensu and the individual likelihoods are combined in the usual way (by multiplication), the final uncertainty can be underestimated, in particular in the case of marginally compatible values.

We correct this effect by adopting the following averaging recipe. The central value is obtained by combining the whole likelihoods. Then we combine the Gaussian uncertainties by combining likelihoods restricted to their Gaussian part. Finally we assign to this combination the smallest of the individual Rfit uncertainties. The underlying idea is twofold:

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates

[1] keeping in mind that in many papers in the literature, this combination is done in quadrature and the splitting between different sources is not published.

- this best estimate should not be penalized by less precise methods (as it would happen be the case if one would take the dispersion of the individual central values as a guess of the combined theoretical uncertainty).

It should be stressed that the concept of a theoretical uncertainty is ill-defined, and the combination of them even more. Thus our approach is only one among the alternatives that can be found in the literature. In contrast to some of the latter, ours is algorithmic and can be reproduced.

2 Decay constants

2.1 Light mesons

f_K

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[1]	2	158.1	0.8	3.1
MILC07	[2]	2+1	156.5	0.4	$^{+1.0}_{-2.7}$
HPQCD07	[3]	2+1	157	0.6	3.3
RBC/UKQCD08	[4]	2+1	149.6	3.6	6.3
Laiho08∗	[5]	2+1	153.9	1.7	6.5
Our average			155.5	0.3	1.9

f_K/f_π

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[1]	2	1.210	0.006	0.024
MILC07	[2]	2+1	1.197	0.003	$^{+0.006}_{-0.013}$
NPLQCD07	[6]	2+1	1.218	0.002	$^{+0.024}_{-0.011}$
HPQCD07	[3]	2+1	1.189	0.002	0.014
RBC/UKQCD08	[4]	2+1	1.205	0.018	0.062
Laiho08∗	[5]	2+1	1.191	0.016	0.026
BMW 2010	[7]	2+1	1.217	0.010	0.019
Our average			1.205	0.0012	0.0095

2.2 Charmed mesons

f_{D_s}

Reference	Article	N_f	Mean	Stat	Syst
CP-PACS00	[8]	2	267	13	$^{+27}_{-17}$
MILC02	[9]	2	241	5	$^{+41}_{-30}$
ETMC09	[1]	2	244	3	9
HPQCD03	[10]	2+1	290	20	64
HPQCD07	[3]	2+1	241	1.4	5.3
FNAL-MILC09$*$	[11]	2+1	260	6.8	14
Our average			246.3	1.2	5.3

f_{D_s}/f_D

Reference	Article	N_f	Mean	Stat	Syst
CP-PACS00	[8]	2	1.182	0.039	$^{+0.087}_{-0.046}$
MILC02	[9]	2	1.14	0.01	$^{+0.06}_{-0.07}$
ETMC09	[1]	2	1.24	0.03	0.01
HPQCD07	[3]	2+1	1.164	0.006	0.020
FNAL-MILC09$*$	[11]	2+1	1.200	0.016	0.025
Our average			1.186	0.0048	0.01

2.3 Beauty mesons

f_{B_s}

Reference	Article	N_f	Mean	Stat	Syst
CP-PACS01	[12]	2	242	9	$^{+53}_{-34}$
MILC02	[9]	2	217	6	$^{+58}_{-31}$
JLQCD03	[13]	2	215	9	$^{+19}_{-15}$
HPQCD03	[10]	2+1	260	7	39
FNAL-MILC09*	[14]	2+1	243	6	22
HPQCD09	[15]	2+1	231	5	30
Our average			228	3	17

f_{B_s}/f_B

Reference	Article	N_f	Mean	Stat	Syst
CP-PACS01	[12]	2	1.179	0.018	0.023
MILC02	[9]	2	1.16	0.01	$^{+0.08}_{-0.04}$
JLQCD03	[13]	2	1.13	0.03	$^{+0.17}_{-0.02}$
FNAL-MILC09*	[14]	2+1	1.245	0.028	0.049
HPQCD09	[15]	2+1	1.226	0.020	0.033
Our average			1.199	0.008	0.023

3 Semileptonic form factors

3.1 $K \to \pi \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
RBC06	[16]	2	0.968	0.009	0.006
ETMC09	[17]	2	0.9560	0.0057	0.0127
RBC-UKQCD07	[18]	2+1	0.9644	0.0033	0.0048
Our average			0.9653	0.0028	0.0048

3.2 $D \to \pi \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
MILC04	[19]	2+1	0.64	0.03	0.15
Our average			0.64	0.03	0.15

3.3 $D \to K \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
MILC04	[19]	2+1	0.73	0.03	0.16
Our average			0.73	0.03	0.16

4 Meson mixing

4.1 Kaon mixing

$B_K^{\bar{\mathrm{MS}}}(2\mathrm{GeV})$

Reference	Article	N_f	Mean	Stat	Syst
JLQCD08	[20]	2	0.537	0.004	0.072
HPQCD/UKQCD06	[21]	2+1	0.618	0.018	0.179
RBC/UKQCD07	[22]	2+1	0.524	0.010	0.052
Laiho09	[23]	2+1	0.527	0.006	0.049
Our average for $B_K^{\mathrm{MS}}(2\mathrm{GeV})$			0.527	0.0031	0.049
Our average for $\hat{B}_K$			0.723	0.0042	0.067

4.2 $B_{d,s}$ mixing

$\hat{B}_{B_s}$

Reference	Article	N_f	Mean	Stat	Syst
JLQCD03	[13]	2	1.299	0.034	$^{+0.122}_{-0.095}$
HPQCD06	[24]	2+1	1.168	0.105	0.140
RBC/UKQCD07*	[25]	2+1	1.21	0.05	0.05
HPQCD09	[15]	2+1	1.326	0.04	0.03
Our average			1.28	0.02	0.03

$\hat{B}_{B_s}/\hat{B}_{B_d}$

Reference	Article	N_f	Mean	Stat	Syst
JLQCD03	[13]	2	1.017	0.016	$^{+0.076}_{-0.017}$
HPQCD09	[15]	2+1	1.053	0.02	0.03
Our average			1.05	0.01	0.03

$f_{B_s}\sqrt{\hat{B}_{B_s}}$ (MeV)

Reference	Article	N_f	Mean	Stat	Syst
JLQCD03	[13]	2	245	10	$^{+24}_{-17}$
HPQCD09	[15]	2+1	266	7	38
Our average			257	6	21

ξ

Reference	Article	N_f	Mean	Stat	Syst
JLQCD03	[13]	2	1.14	0.03	$^{+0.17}_{-0.02}$
HPQCD09	[15]	2+1	1.258	0.026	0.043
Our average			1.258	0.020	0.043

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