

# UPDATED RESULTS ON THE CKM MATRIX AND THE UNITARITY TRIANGLE

Including results presented up to  
LEPTON-PHOTON 11

*P r e l i m i n a r y*

October 1st, 2011

The CKMfitter Group

## Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach *Rfit*. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

*CP* VIOLATION AND THE CKM MATRIX:  
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES

By CKMfitter Group

Eur. Phys. J. **C41**, 1-131, 2005 [hep-ph/0406184]

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## 1 Inputs

| Parameter                                                                                                 | Value $\pm$ Error(s)                       | Reference | Errors |    |
|-----------------------------------------------------------------------------------------------------------|--------------------------------------------|-----------|--------|----|
|                                                                                                           |                                            |           | GS     | TH |
| $ V_{ud} $ (nuclei)                                                                                       | $0.97425 \pm 0.00022$                      | [1]       | ★      | -  |
| $ V_{us} $ ( $K_{\ell 3}$ )                                                                               | $0.2246 \pm 0.0009 \pm 0.0012$             | [2, 3]    | ★      | ★  |
| $ V_{ub} $                                                                                                | $(3.92 \pm 0.09 \pm 0.45) \times 10^{-3}$  | [4, 5]    | ★      | ★  |
| $ V_{cb} $                                                                                                | $(40.89 \pm 0.38 \pm 0.59) \times 10^{-3}$ | [4]       | ★      | ★  |
| $\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau)$                                                      | $(1.68 \pm 0.31) \times 10^{-4}$           | [6]       | ★      | -  |
| $\mathcal{B}(D_s^- \rightarrow \mu^- \bar{\nu}_\mu)$                                                      | $(5.90 \pm 0.33) \times 10^{-3}$           | [4]       | ★      | -  |
| $\mathcal{B}(D_s^- \rightarrow \tau^- \bar{\nu}_\tau)$                                                    | $(5.29 \pm 0.28) \times 10^{-2}$           | [4]       | ★      | -  |
| $\mathcal{B}(D^- \rightarrow \mu^- \bar{\nu}_\mu)$                                                        | $(3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$  | [7]       | ★      | ★  |
| $\mathcal{B}(K^- \rightarrow e^- \bar{\nu}_e)$                                                            | $(1.584 \pm 0.020) \times 10^{-5}$         | [9]       | ★      | -  |
| $\mathcal{B}(K^- \rightarrow \mu^- \bar{\nu}_\mu)$                                                        | $0.6347 \pm 0.0018$                        | [2]       | ★      | -  |
| $\mathcal{B}(\tau^- \rightarrow K^- \bar{\nu}_\tau)$                                                      | $(0.696 \pm 0.023) \times 10^{-3}$         | [9]       | ★      | -  |
| $\mathcal{B}(K^- \rightarrow \mu^- \bar{\nu}_\mu)/\mathcal{B}(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$     | $1.344 \pm 0.0041$                         | [2]       | ★      | -  |
| $\mathcal{B}(\tau^- \rightarrow K^- \bar{\nu}_\tau)/\mathcal{B}(\tau^- \rightarrow \pi^- \bar{\nu}_\tau)$ | $(6.53 \pm 0.11) \times 10^{-2}$           | [8]       | ★      | -  |
| $ \varepsilon_K $                                                                                         | $(2.229 \pm 0.010) \times 10^{-3}$         | [9]       | ★      | -  |
| $\Delta m_d$                                                                                              | $(0.507 \pm 0.005) \text{ ps}^{-1}$        | [4]       | ★      | -  |
| $\Delta m_s$                                                                                              | $(17.731 \pm 0.045) \text{ ps}^{-1}$       | [10]      | ★      | -  |
| $\sin(2\beta)_{[c\bar{c}]}$                                                                               | $0.679 \pm 0.020$                          | [4]       | ★      | -  |
| $S_{\pi\pi}^{+-}, C_{\pi\pi}^{+-}, C_{\pi\pi}^{00}$                                                       | Inputs to isospin analysis                 | [4]       | ★      | -  |
| $\mathcal{B}_{\pi\pi}$ all charges                                                                        | Inputs to isospin analysis                 | [4]       | ★      | -  |
| $S_{\rho\rho,L}^{+-}, C_{\rho\rho,L}^{+-}, S_{\rho\rho}^{00}, C_{\rho\rho}^{00}$                          | Inputs to isospin analysis                 | [4]       | ★      | -  |
| $\mathcal{B}_{\rho\rho,L}$ all charges                                                                    | Inputs to isospin analysis                 | [4]       | ★      | -  |
| $B^0 \rightarrow (\rho\pi)^0 \rightarrow 3\pi$                                                            | Time-dependent Dalitz analysis             | [11, 12]  | ★      | -  |
| $B^- \rightarrow D^{(*)} K^{(*)-}$                                                                        | Inputs to GLW analysis                     | [4]       | ★      | -  |
| $B^- \rightarrow D^{(*)} K^{(*)-}$                                                                        | Inputs to ADS analysis                     | [4]       | ★      | -  |
| $B^- \rightarrow D^{(*)} K^{(*)-}$                                                                        | GSZ Dalitz analysis                        | [4]       | ★      | -  |

Table 1: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the “GS” column by an asterisk; measurements or parameters that have systematic theoretical errors are marked in the “TH” column by an asterisk. Upper part: experimental determinations of the CKM matrix elements. Lower part: CP-violation and mixing observables.*

| Parameter                      | Value $\pm$ Error(s)                                 | Reference | Errors |    |
|--------------------------------|------------------------------------------------------|-----------|--------|----|
|                                |                                                      |           | GS     | TH |
| $\overline{m}_c(m_c)$          | $(1.286 \pm 0.013 \pm 0.040)$ GeV                    | [13]      | *      | *  |
| $\overline{m}_t(m_t)$          | $(165.02 \pm 1.16 \pm 0.11)$ GeV                     | [17]      | *      | *  |
| $\alpha_s(m_Z^2)$              | $0.1184 \pm 0.0007$                                  | [9]       | -      | *  |
| $B_K$                          | $0.732 \pm 0.004 \pm 0.036$                          | [3]       | *      | *  |
| $\kappa_\epsilon$              | $0.940 \pm 0.013 \pm 0.023$                          | [19]      | *      | *  |
| $\eta_{cc}$                    | Calculated from $\overline{m}_c(m_c)$ and $\alpha_s$ | [20]      | -      | *  |
| $\eta_{ct}$                    | $0.47 \pm 0.04$                                      | [21]      | -      | *  |
| $\eta_{tt}$                    | $0.5765 \pm 0.0065$                                  | [20, 21]  | -      | *  |
| $\eta_B(\overline{\text{MS}})$ | $0.5510 \pm 0.0022$                                  | [22, 23]  | -      | *  |
| $f_{B_s}$                      | $(231 \pm 3 \pm 15)$ MeV                             | [3]       | *      | *  |
| $B_s$                          | $1.291 \pm 0.025 \pm 0.035$                          | [3]       | *      | *  |
| $f_{B_s}/f_{B_d}$              | $1.235 \pm 0.008 \pm 0.033$                          | [3]       | *      | *  |
| $B_s/B_d$                      | $1.024 \pm 0.013 \pm 0.015$                          | [3]       | *      | *  |
| $f_K$                          | $(156.3 \pm 0.3 \pm 1.9)$ MeV                        | [3]       | *      | *  |
| $f_K/f_\pi$                    | $1.198 \pm 0.002 \pm 0.010$                          | [3]       | *      | *  |
| $f_{D_S}$                      | $(249 \pm 2 \pm 5)$ MeV                              | [3]       | *      | *  |
| $f_{D_S}/f_D$                  | $1.185 \pm 0.005 \pm 0.010$                          | [3]       | *      | *  |

Table 2: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the “GS” column by an asterisk; measurements or parameters that have systematic theoretical errors are marked in the “TH” column by an asterisk. Upper part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

## 2 Lattice QCD averages

Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

### 2.1 Method of averaging

We collect the relevant calculations of the quantity that we are interested in: we take only unquenched results with 2 or 2+1 dynamical fermions, even those from proceedings without a companion article. In these results, we separate the error estimates into a Gaussian part and a flat part that is treated à la Rfit. The Gaussian part collects the uncertainties from purely statistical origin, but also the systematics that can be controlled and treated in a similar way (e.g., interpolation or fitting in some cases). The remaining systematics constitute the Rfit error. If there are several sources of error in the Rfit category, we add them linearly <sup>1</sup>.

The Rfit model is simple but also very strict. It amounts to assuming that the theoretical uncertainty is rigorously constrained by a mathematical bound that is our only piece of information. If Rfit is taken *stricto sensu* and the individual likelihoods are combined in the usual way (by multiplication), the final uncertainty can be underestimated, in particular in the case of marginally compatible values.

We correct this effect by adopting the following averaging recipe. The central value is obtained by combining the whole likelihoods. Then we combine the Gaussian uncertainties by combining likelihoods restricted to their Gaussian part. Finally we assign to this combination the smallest of the individual Rfit uncertainties. The underlying idea is twofold:

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
- this best estimate should not be penalized by less precise methods (as it would happen be the case if one would take the dispersion of the individual central values as a guess of the combined theoretical uncertainty).

It should be stressed that the concept of a theoretical uncertainty is ill-defined, and the combination of them even more. Thus our approach is only one among the alternatives that can be found in the literature. In contrast to some of the latter, ours is algorithmic and can be reproduced.

### 2.2 Decay constants

#### 2.2.1 Light mesons

$f_K$

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<sup>1</sup>keeping in mind that in many papers in the literature, this combination is done in quadrature and the splitting between different sources is not published.

| Reference   | Article | $N_f$ | Mean  | Stat | Syst             |
|-------------|---------|-------|-------|------|------------------|
| ETMC09      | [24]    | 2     | 158.1 | 0.8  | 3.1              |
| MILC07      | [25]    | 2+1   | 156.5 | 0.4  | $+1.0$<br>$-2.7$ |
| HPQCD07     | [26]    | 2+1   | 157   | 0.4  | 3.3              |
| ALVdW08     | [27]    | 2+1   | 153.9 | 1.7  | 6.5              |
| Our average |         |       | 156.3 | 0.3  | 1.9              |

$f_K/f_\pi$

| Reference   | Article | $N_f$ | Mean   | Stat   | Syst                 |
|-------------|---------|-------|--------|--------|----------------------|
| ETMC09      | [24]    | 2     | 1.210  | 0.006  | 0.024                |
| MILC07      | [25]    | 2+1   | 1.197  | 0.003  | $+0.006$<br>$-0.013$ |
| NPLQCD07    | [28]    | 2+1   | 1.218  | 0.002  | $+0.011$<br>$-0.024$ |
| HPQCD07     | [26]    | 2+1   | 1.189  | 0.002  | 0.014                |
| ALVdW08     | [27]    | 2+1   | 1.191  | 0.016  | 0.026                |
| BMW10       | [29]    | 2+1   | 1.192  | 0.007  | 0.013                |
| Our average |         |       | 1.1985 | 0.0013 | 0.0095               |

## 2.2.2 Charmed mesons

$f_{D_s}$

| Reference   | Article | $N_f$ | Mean  | Stat | Syst           |
|-------------|---------|-------|-------|------|----------------|
| MILC02      | [30]    | 2     | 241   | 5    | $+41$<br>$-30$ |
| ETMC09      | [24]    | 2     | 244   | 3    | 9              |
| HPQCD03     | [31]    | 2+1   | 290   | 20   | 64             |
| FNAL-MILC09 | [32]    | 2+1   | 260   | 6.8  | 14             |
| HPQCD10     | [33]    | 2+1   | 248.0 | 1.4  | 4.5            |
| Our average |         |       | 249.2 | 1.2  | 4.5            |

$f_{D_s}/f_D$

| Reference   | Article | $N_f$ | Mean  | Stat  | Syst               |
|-------------|---------|-------|-------|-------|--------------------|
| MILC02      | [30]    | 2     | 1.14  | 0.01  | $+0.06$<br>$-0.07$ |
| ETMC09      | [24]    | 2     | 1.24  | 0.03  | 0.01               |
| HPQCD07     | [26]    | 2+1   | 1.164 | 0.005 | 0.020              |
| FNAL-MILC09 | [32]    | 2+1   | 1.200 | 0.016 | 0.025              |
| Our average |         |       | 1.185 | 0.005 | 0.010              |

## 2.2.3 Beauty mesons

$f_{B_s}$

| Reference   | Article | $N_f$ | Mean | Stat | Syst           |
|-------------|---------|-------|------|------|----------------|
| MILC02      | [30]    | 2     | 217  | 6    | $^{+58}_{-31}$ |
| JLQCD03     | [34]    | 2     | 215  | 9    | $^{+19}_{-15}$ |
| ETMC09      | [35]    | 2     | 243  | 6    | 15             |
| HPQCD03     | [31]    | 2+1   | 260  | 7    | 39             |
| FNAL-MILC09 | [36]    | 2+1   | 243  | 6    | 23             |
| HPQCD09     | [37]    | 2+1   | 231  | 5    | 30             |
| Our average |         |       | 231  | 3    | 15             |

$f_{B_s}/f_B$

| Reference   | Article | $N_f$ | Mean  | Stat  | Syst               |
|-------------|---------|-------|-------|-------|--------------------|
| MILC02      | [30]    | 2     | 1.16  | 0.01  | $^{+0.08}_{-0.04}$ |
| JLQCD03     | [34]    | 2     | 1.13  | 0.03  | $^{+0.17}_{-0.02}$ |
| ETMC09      | [35]    | 2     | 1.27  | 0.03  | 0.04               |
| FNAL-MILC09 | [36]    | 2+1   | 1.245 | 0.028 | 0.049              |
| HPQCD09     | [37]    | 2+1   | 1.226 | 0.020 | 0.033              |
| RBC/UKQCD10 | [38]    | 2+1   | 1.15  | 0.05  | 0.20               |
| Our average |         |       | 1.235 | 0.008 | 0.033              |

## 2.3 Semileptonic form factors

### 2.3.1 $K \rightarrow \pi \ell \nu$

$f_+(0)$

| Reference   | Article | $N_f$ | Mean   | Stat   | Syst                   |
|-------------|---------|-------|--------|--------|------------------------|
| RBC06       | [39]    | 2     | 0.968  | 0.009  | 0.006                  |
| ETMC09      | [40]    | 2     | 0.9560 | 0.0057 | 0.0128                 |
| RBC-UKQCD10 | [41]    | 2+1   | 0.9599 | 0.0034 | $^{+0.0045}_{-0.0057}$ |
| Our average |         |       | 0.9632 | 0.0028 | 0.0051                 |

Combining with  $|V_{us}|f_+(0) = 0.2163(5)$  from ref.[42], we get  $|V_{us}| = 0.2246 \pm 0.0009 \pm 0.0012$

### 2.3.2 $D \rightarrow \pi \ell \nu$

$f_+(0)$

| Reference   | Article | $N_f$ | Mean | Stat | Syst |
|-------------|---------|-------|------|------|------|
| ETMC11      | [43]    | 2     | 0.65 | 0.06 | 0.12 |
| MILC04      | [44]    | 2+1   | 0.64 | 0.03 | 0.15 |
| Our average |         |       | 0.65 | 0.03 | 0.12 |

### 2.3.3 $D \rightarrow K \ell \nu$

$f_+(0)$

| Reference   | Article | $N_f$ | Mean  | Stat  | Syst  |
|-------------|---------|-------|-------|-------|-------|
| ETMC11      | [43]    | 2     | 0.76  | 0.05  | 0.11  |
| MILC04      | [44]    | 2+1   | 0.73  | 0.03  | 0.16  |
| HPQCD10     | [45]    | 2+1   | 0.747 | 0.011 | 0.034 |
| Our average |         |       | 0.747 | 0.010 | 0.034 |

## 2.4 Meson mixing

### 2.4.1 Kaon mixing

$B_K^{\text{MS}}(2\text{GeV})$

| Reference                                      | Article | $N_f$ | Mean  | Stat  | Syst  |
|------------------------------------------------|---------|-------|-------|-------|-------|
| JLQCD08                                        | [46]    | 2     | 0.537 | 0.004 | 0.072 |
| ETMC 10                                        | [47]    | 2     | 0.532 | 0.019 | 0.026 |
| HPQCD/UKQCD06                                  | [48]    | 2+1   | 0.618 | 0.018 | 0.179 |
| ALVdW09                                        | [49]    | 2+1   | 0.527 | 0.006 | 0.035 |
| RBC/UKQCD10                                    | [50]    | 2+1   | 0.549 | 0.005 | 0.038 |
| BSW10                                          | [51]    | 2+1   | 0.523 | 0.007 | 0.039 |
| Our average for $B_K^{\text{MS}}(2\text{GeV})$ |         |       | 0.534 | 0.003 | 0.026 |
| Our average for $\hat{B}_K$                    |         |       | 0.732 | 0.004 | 0.036 |

The central value in ref. [51] was diminished by 0.6% due to finite-volume effects, following the discussion in ref. [52].

### 2.4.2 $B_{d,s}$ mixing

$\hat{B}_{B_s}$

| Reference   | Article | $N_f$ | Mean  | Stat  | Syst                 |
|-------------|---------|-------|-------|-------|----------------------|
| JLQCD03     | [34]    | 2     | 1.299 | 0.034 | $^{+0.122}_{-0.087}$ |
| HPQCD06     | [53]    | 2+1   | 1.187 | 0.086 | 0.108                |
| HPQCD09     | [37]    | 2+1   | 1.322 | 0.040 | 0.035                |
| Our average |         |       | 1.291 | 0.025 | 0.035                |

Ref. [37] provide only  $f_{B_s}$  and  $f_{B_s}\sqrt{\hat{B}_{B_s}}$ , and we assumed that the systematics were completely correlated to extract  $\hat{B}_{B_s}$ .

$\hat{B}_{B_s}/\hat{B}_{B_d}$

| Reference   | Article | $N_f$ | Mean  | Stat  | Syst                 |
|-------------|---------|-------|-------|-------|----------------------|
| JLQCD03     | [34]    | 2     | 1.017 | 0.016 | $^{+0.076}_{-0.017}$ |
| HPQCD09     | [37]    | 2+1   | 1.052 | 0.027 | 0.015                |
| RBC/UKQCD10 | [38]    | 2+1   | 0.959 | 0.038 | 0.040                |
| Our average |         |       | 1.024 | 0.013 | 0.015                |

Refs. [37] and [38] provide only  $\xi$  and  $f_{B_s}/f_{B_d}$ . We have extracted  $\hat{B}_{B_s}/\hat{B}_{B_d}$  in both cases assuming a total correlation in the systematics of  $\xi$  and  $\hat{B}_{B_s}/\hat{B}_{B_d}$ .



### 3 Results

| Observable                                                                      | central $\pm$ CL $\equiv 1\sigma$ | $\pm$ CL $\equiv 2\sigma$                 | $\pm$ CL $\equiv 3\sigma$           |
|---------------------------------------------------------------------------------|-----------------------------------|-------------------------------------------|-------------------------------------|
| $A$                                                                             | $0.801^{+0.026}_{-0.014}$         | $0.801^{+0.036}_{-0.022}$                 | $0.801^{+0.046}_{-0.029}$           |
| $\lambda$                                                                       | $0.22539^{+0.00062}_{-0.00095}$   | $0.2254^{+0.0010}_{-0.0019}$              | $0.2254^{+0.0014}_{-0.0027}$        |
| $\bar{\rho}$                                                                    | $0.144^{+0.023}_{-0.026}$         | $0.144^{+0.038}_{-0.046}$                 | $0.144^{+0.048}_{-0.057}$           |
| $\bar{\eta}$                                                                    | $0.343^{+0.015}_{-0.014}$         | $0.343^{+0.030}_{-0.025}$                 | $0.343^{+0.045}_{-0.033}$           |
| $J$ [ $10^{-5}$ ]                                                               | $2.884^{+0.253}_{-0.053}$         | $2.884^{+0.400}_{-0.098}$                 | $2.88^{+0.55}_{-0.14}$              |
| $\sin 2\alpha$                                                                  | $-0.03^{+0.14}_{-0.12}$           | $-0.03^{+0.25}_{-0.21}$                   | $-0.03^{+0.31}_{-0.26}$             |
| $\sin 2\alpha$ (meas. not in the fit)                                           | $-0.10^{+0.18}_{-0.12}$           | $-0.10^{+0.34}_{-0.17}$                   | $-0.10^{+0.40}_{-0.22}$             |
| $\sin 2\beta$                                                                   | $0.691^{+0.020}_{-0.020}$         | $0.691^{+0.040}_{-0.034}$                 | $0.691^{+0.060}_{-0.047}$           |
| $\sin 2\beta$ (meas. not in the fit)                                            | $0.830^{+0.013}_{-0.033}$         | $0.830^{+0.025}_{-0.098}$                 | $0.830^{+0.037}_{-0.170}$           |
| $\alpha$ [ $^\circ$ ]                                                           | $90.9^{+3.5}_{-4.1}$              | $90.9^{+6.0}_{-7.2}$                      | $90.9^{+7.5}_{-8.9}$                |
| $\alpha$ [ $^\circ$ ] (meas. not in the fit)                                    | $92.9^{+3.6}_{-5.1}$              | $92.9^{+5.0}_{-9.8}$                      | $92.9^{+6.4}_{-11.7}$               |
| $\alpha$ [ $^\circ$ ] (dir. meas.)                                              | $89.0^{+4.4}_{-4.2}$              | $89.0^{+9.1}_{-8.4}    - 2^{+6.6}_{-5.6}$ | $89^{+21}_{-13}    - 2^{+14}_{-14}$ |
| $\beta$ [ $^\circ$ ]                                                            | $21.84^{+0.80}_{-0.76}$           | $21.8^{+1.6}_{-1.3}$                      | $21.8^{+2.5}_{-1.8}$                |
| $\beta$ [ $^\circ$ ] (meas. not in the fit)                                     | $28.06^{+0.67}_{-1.62}$           | $28.1^{+1.4}_{-4.5}$                      | $28.1^{+2.0}_{-7.4}$                |
| $\beta$ [ $^\circ$ ] (dir. meas.)                                               | $21.38^{+0.79}_{-0.77}$           | $21.4^{+1.6}_{-1.5}$                      | $21.4^{+2.4}_{-2.3}$                |
| $\gamma$ [ $^\circ$ ]                                                           | $67.3^{+4.2}_{-3.5}$              | $67.3^{+7.2}_{-5.7}$                      | $67.3^{+8.9}_{-7.1}$                |
| $\gamma$ [ $^\circ$ ] (meas. not in the fit)                                    | $67.1^{+4.6}_{-3.7}$              | $67.1^{+7.5}_{-5.7}$                      | $67.1^{+9.2}_{-7.0}$                |
| $\gamma$ [ $^\circ$ ] (dir. meas.)                                              | $68^{+10}_{-11}$                  | $68^{+21}_{-21}$                          | $68^{+35}_{-31}$                    |
| $R_u$                                                                           | $0.372^{+0.013}_{-0.012}$         | $0.372^{+0.026}_{-0.021}$                 | $0.372^{+0.040}_{-0.029}$           |
| $R_t$                                                                           | $0.923^{+0.027}_{-0.023}$         | $0.923^{+0.046}_{-0.037}$                 | $0.923^{+0.057}_{-0.047}$           |
| $\bar{\rho}_s$                                                                  | $-0.0077^{+0.0014}_{-0.0012}$     | $-0.0077^{+0.0025}_{-0.0020}$             | $-0.0077^{+0.0031}_{-0.0026}$       |
| $\bar{\eta}_s$                                                                  | $-0.01838^{+0.00075}_{-0.00080}$  | $-0.0184^{+0.0013}_{-0.0016}$             | $-0.0184^{+0.0018}_{-0.0024}$       |
| $\beta_s \equiv -\arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right)$ [rad] | $0.01824^{+0.00080}_{-0.00075}$   | $0.0182^{+0.0016}_{-0.0013}$              | $0.0182^{+0.0024}_{-0.0018}$        |
| $\sin 2\beta_s$                                                                 | $0.0365^{+0.0016}_{-0.0015}$      | $0.0365^{+0.0032}_{-0.0027}$              | $0.0365^{+0.0049}_{-0.0035}$        |

| Observable                                                                   | central $\pm$ CL $\equiv 1\sigma$  | $\pm$ CL $\equiv 2\sigma$          | $\pm$ CL $\equiv 3\sigma$          |
|------------------------------------------------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| $ V_{ud} $                                                                   | $0.97426^{+0.00022}_{-0.00014}$    | $0.97426^{+0.00044}_{-0.00024}$    | $0.97426^{+0.00063}_{-0.00032}$    |
| $ V_{us} $                                                                   | $0.22539^{+0.00062}_{-0.00095}$    | $0.2254^{+0.0010}_{-0.0019}$       | $0.2254^{+0.0014}_{-0.0027}$       |
| $ V_{ub} $                                                                   | $0.003501^{+0.000196}_{-0.000087}$ | $0.00350^{+0.00033}_{-0.00016}$    | $0.00350^{+0.00046}_{-0.00022}$    |
| $ V_{cd} $                                                                   | $0.22526^{+0.00062}_{-0.00095}$    | $0.2253^{+0.0010}_{-0.0019}$       | $0.2253^{+0.0014}_{-0.0027}$       |
| $ V_{cs} $                                                                   | $0.97345^{+0.00022}_{-0.00018}$    | $0.97345^{+0.00043}_{-0.00027}$    | $0.97345^{+0.00063}_{-0.00035}$    |
| $ V_{cb} $                                                                   | $0.04070^{+0.00116}_{-0.00059}$    | $0.04070^{+0.00154}_{-0.00095}$    | $0.0407^{+0.0019}_{-0.0013}$       |
| $ V_{td} $                                                                   | $0.00846^{+0.00043}_{-0.00015}$    | $0.00846^{+0.00065}_{-0.00028}$    | $0.00846^{+0.00078}_{-0.00038}$    |
| $ V_{ts} $                                                                   | $0.03996^{+0.00114}_{-0.00062}$    | $0.03996^{+0.00152}_{-0.00098}$    | $0.0400^{+0.0019}_{-0.0013}$       |
| $ V_{tb} $                                                                   | $0.999165^{+0.000024}_{-0.000048}$ | $0.999165^{+0.000038}_{-0.000064}$ | $0.999165^{+0.000051}_{-0.000081}$ |
| $ V_{ud} $ (meas. not in the fit)                                            | $0.97473^{+0.00021}_{-0.00062}$    | $0.97473^{+0.00041}_{-0.00073}$    | $0.97473^{+0.00061}_{-0.00084}$    |
| $ V_{us} $ (meas. not in the fit)                                            | $0.22539^{+0.00094}_{-0.00095}$    | $0.2254^{+0.0019}_{-0.0019}$       | $0.2254^{+0.0028}_{-0.0029}$       |
| $ V_{ub} $ (meas. not in the fit)                                            | $0.00342^{+0.00020}_{-0.00010}$    | $0.00342^{+0.00033}_{-0.00020}$    | $0.00342^{+0.00047}_{-0.00029}$    |
| $ V_{cb} $ (meas. not in the fit)                                            | $0.04070^{+0.00446}_{-0.00069}$    | $0.0407^{+0.0050}_{-0.0013}$       | $0.0407^{+0.0055}_{-0.0018}$       |
| $\Delta m_d$ [ps $^{-1}$ ] (meas. not in the fit)                            | $0.537^{+0.051}_{-0.043}$          | $0.537^{+0.087}_{-0.103}$          | $0.54^{+0.13}_{-0.14}$             |
| $\Delta m_s$ [ps $^{-1}$ ] (meas. not in the fit)                            | $18.1^{+2.2}_{-2.1}$               | $18.1^{+3.5}_{-3.2}$               | $18.1^{+4.6}_{-4.1}$               |
| $ \epsilon_K $ [10 $^{-3}$ ] (meas. not in the fit)                          | $1.86^{+0.67}_{-0.39}$             | $1.86^{+0.84}_{-0.47}$             | $1.86^{+1.00}_{-0.54}$             |
| $m_c$ [GeV/c $^2$ ] (meas. not in the fit)                                   | $1.62^{+0.31}_{-0.59}$             | $1.62^{+0.39}_{-0.78}$             | $1.62^{+0.45}_{-0.98}$             |
| $m_t$ [GeV/c $^2$ ] (meas. not in the fit)                                   | $153.6^{+7.6}_{-7.4}$              | $154^{+31}_{-12}$                  | $154^{+46}_{-15}$                  |
| $B_K$ (lattice value not in the fit)                                         | $0.89^{+0.16}_{-0.22}$             | $0.89^{+0.22}_{-0.26}$             | $0.89^{+0.28}_{-0.29}$             |
| $\xi_{\text{SU}(3)}$ (lattice value not in the fit)                          | $1.218^{+0.048}_{-0.042}$          | $1.218^{+0.092}_{-0.083}$          | $1.22^{+0.13}_{-0.12}$             |
| $f_{B_s}$ (lattice value not in the fit)                                     | $0.2385^{+0.0048}_{-0.0127}$       | $0.2385^{+0.0086}_{-0.0175}$       | $0.238^{+0.012}_{-0.021}$          |
| $\mathcal{B}(B^+ \rightarrow \tau \nu)$ [10 $^{-4}$ ]                        | $0.832^{+0.084}_{-0.084}$          | $0.83^{+0.15}_{-0.18}$             | $0.83^{+0.23}_{-0.22}$             |
| $\mathcal{B}(B^+ \rightarrow \tau \nu)$ [10 $^{-4}$ ] (meas. not in the fit) | $0.739^{+0.114}_{-0.087}$          | $0.74^{+0.20}_{-0.12}$             | $0.74^{+0.27}_{-0.15}$             |
| $\mathcal{B}(B^+ \rightarrow \mu \nu)$ [10 $^{-6}$ ]                         | $0.374^{+0.038}_{-0.038}$          | $0.374^{+0.070}_{-0.081}$          | $0.374^{+0.102}_{-0.099}$          |
| $\mathcal{B}(B^+ \rightarrow e \nu)$ [10 $^{-11}$ ]                          | $0.875^{+0.088}_{-0.089}$          | $0.87^{+0.16}_{-0.19}$             | $0.87^{+0.24}_{-0.23}$             |
| $\mathcal{B}(B_d \rightarrow e^+ e^-)$ [10 $^{-15}$ ]                        | $2.61^{+0.14}_{-0.24}$             | $2.61^{+0.22}_{-0.38}$             | $2.61^{+0.30}_{-0.45}$             |
| $\mathcal{B}(B_d \rightarrow \mu^+ \mu^-)$ [10 $^{-11}$ ]                    | $11.13^{+0.61}_{-1.01}$            | $11.13^{+0.94}_{-1.64}$            | $11.1^{+1.3}_{-1.9}$               |
| $\mathcal{B}(B_s \rightarrow e^+ e^-)$ [10 $^{-14}$ ]                        | $8.49^{+0.43}_{-0.76}$             | $8.49^{+0.64}_{-1.00}$             | $8.49^{+0.86}_{-1.17}$             |
| $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ [10 $^{-9}$ ]                     | $3.63^{+0.18}_{-0.32}$             | $3.63^{+0.27}_{-0.43}$             | $3.63^{+0.37}_{-0.50}$             |

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- [5] For the inclusive average we are taking the BLNP number. (The DGE result is very close to the BLNP result. The uncertainties between BLNP and DGE are hard to compare.) The theoretical error on the inclusive average is obtained by adding linearly the contributions from weak annihilation, subleading shape functions and the HQE uncertainty on  $m_b$ . We use only branching fractions measured for  $B \rightarrow \pi \ell \nu$ . We average the results obtained from the two unquenched Lattice calculations and the LCSR calculation for the form factor quoted by HFAG [4] in such a way that the smallest theoretical error is kept. We keep the smallest theoretical error also for the average between the inclusive and exclusive results.
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This input is consistent with  $\overline{m}_c(m_c) = (1.265 \pm 0.060 \pm 0.050)$  GeV translated from the kinetic charm mass obtained from fits to data from lepton energy and hadronic mass moments in  $B \rightarrow X_c \ell \nu$  decays combined with photon energy moments measured in  $B \rightarrow X_s \gamma$  decays:  $m_c^{kin} = (1.165 \pm 0.050)$  GeV [4].

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