

UPDATED RESULTS ON THE CKM MATRIX

Including results presented up to
Moriond 14

P r e l i m i n a r y

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The CKMfitter Group

Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach *Rfit*. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

CP VIOLATION AND THE CKM MATRIX:
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES

By CKMfitter Group

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1 Inputs

Parameter	Value \pm Error(s)	Reference	Errors	
			GS	TH
$ V_{ud} $ (nuclei)	$0.97425 \pm 0 \pm 0.00022$	[1]	-	*
$ V_{us} $ ($K_{\ell 3}$)	$0.2247 \pm 0.0006 \pm 0.0011$	[4, 3]	*	*
$ V_{cd} $ (νN)	0.230 ± 0.011	[4]	*	-
$ V_{cs} $ ($W \rightarrow c\bar{s}$)	$0.94^{+0.32}_{-0.26} \pm 0.13$	[4]	*	*
$ V_{ub} $	$(3.70 \pm 0.12 \pm 0.26) \times 10^{-3}$	[5, 6]	*	*
$ V_{cb} $	$(41.00 \pm 0.33 \pm 0.74) \times 10^{-3}$	[5]	*	*
$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau)$	$(1.14 \pm 0.22) \times 10^{-4}$	[7, 5]	*	-
$\mathcal{B}(D_s^- \rightarrow \mu^- \bar{\nu}_\mu)$	$(5.54 \pm 0.24) \times 10^{-3}$	[5]	*	-
$\mathcal{B}(D_s^- \rightarrow \tau^- \bar{\nu}_\tau)$	$(5.44 \pm 0.22) \times 10^{-2}$	[5]	*	-
$\mathcal{B}(D^- \rightarrow \mu^- \bar{\nu}_\mu)$	$(3.74 \pm 0.17) \times 10^{-4}$	[8, 9]	*	*
$\mathcal{B}(K^- \rightarrow e^- \bar{\nu}_e)$	$(1.581 \pm 0.008) \times 10^{-5}$	[4]	*	-
$\mathcal{B}(K^- \rightarrow \mu^- \bar{\nu}_\mu)$	0.6355 ± 0.0011	[2]	*	-
$\mathcal{B}(\tau^- \rightarrow K^- \bar{\nu}_\tau)$	$(0.700 \pm 0.010) \times 10^{-3}$	[4]	*	-
$\mathcal{B}(K^- \rightarrow \mu^- \bar{\nu}_\mu) / \mathcal{B}(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$	1.344 ± 0.0041	[2]	*	-
$\mathcal{B}(\tau^- \rightarrow K^- \bar{\nu}_\tau) / \mathcal{B}(\tau^- \rightarrow \pi^- \bar{\nu}_\tau)$	$(6.53 \pm 0.11) \times 10^{-2}$	[10]	*	-
$ V_{cd} f_+^{D \rightarrow \pi}(0)$	0.148 ± 0.004	[11]	*	-
$ V_{cs} f_+^{D \rightarrow K}(0)$	0.712 ± 0.007	[11, 12]	*	-
$ \varepsilon_K $	$(2.228 \pm 0.011) \times 10^{-3}$	[4]	*	-
Δm_d	$(0.507 \pm 0.004) \text{ ps}^{-1}$	[5]	*	-
Δm_s	$(17.762 \pm 0.023) \text{ ps}^{-1}$	[13, 5]	*	-
$\sin(2\beta)_{[c\bar{c}]}$	0.682 ± 0.018	[5]	*	-
$(\beta_s)_{J/\psi\phi}$	$-0.013^{+0.083}_{-0.090}$	[5]	*	-
$S_{\pi\pi}^{+-}, C_{\pi\pi}^{+-}, C_{\pi\pi}^{00}, \mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis	[14]	*	-
$S_{\rho\rho,L}^{+-}, C_{\rho\rho,L}^{+-}, S_{\rho\rho}^{00}, C_{\rho\rho}^{00}, \mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[15]	*	-
$B_0^0 \rightarrow (\rho\pi)_0^0 \rightarrow 3\pi$	Time-dependent Dalitz analysis	[16]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to GLW analysis	[17]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	Inputs to ADS analysis	[18]	*	-
$B^- \rightarrow D^{(*)} K^{(*)-}$	GGSZ Dalitz analysis	[19]	*	-

Table 1: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the ‘‘GS’’ column by a star; measurements or parameters that have systematic theoretical errors are marked in the ‘‘TH’’ column by a star. Upper part: experimental determinations of the CKM matrix elements. Lower part: CP-violation and mixing observables.*

Parameter	Value \pm Error(s)	Reference	Errors	
			GS	TH
$\overline{m}_c(m_c)$	$(1.286 \pm 0.013 \pm 0.040)$ GeV	[20]	*	*
$\overline{m}_t(m_t)$	$(165.95 \pm 0.35 \pm 0.64)$ GeV	[24]	*	*
$\alpha_s(m_Z)$	$0.1185 \pm 0 \pm 0.0006$	[4]	-	*
B_K	$0.7615 \pm 0.0027 \pm 0.0137$	[3]	*	*
κ_ϵ	$0.940 \pm 0.013 \pm 0.023$	[26, 27]	*	*
η_{cc}	$1.87 \pm 0 \pm 0.76$	[28]	-	*
η_{ct}	$0.497 \pm 0 \pm 0.047$	[29]	-	*
η_{tt}	$0.5765 \pm 0 \pm 0.0065$	[30]	-	*
$\eta_B(\overline{\text{MS}})$	$0.5510 \pm 0 \pm 0.0022$	[31, 32]	-	*
f_{B_s}	$(225.6 \pm 1.1 \pm 5.4)$ MeV	[3]	*	*
B_s	$1.320 \pm 0.017 \pm 0.030$	[3]	*	*
f_{B_s}/f_{B_d}	$1.205 \pm 0.004 \pm 0.007$	[3]	*	*
B_s/B_d	$1.023 \pm 0.013 \pm 0.014$	[3]	*	*
f_K	$(155.2 \pm 0.2 \pm 0.6)$ MeV	[3]	*	*
f_K/f_π	$1.1942 \pm 0.0009 \pm 0.0030$	[3]	*	*
f_{D_S}	$(245.3 \pm 0.5 \pm 4.5)$ MeV	[3]	*	*
f_{D_S}/f_D	$1.201 \pm 0.004 \pm 0.010$	[3]	*	*
$f_+^{D \rightarrow \pi}(0)$	$0.666 \pm 0.020 \pm 0.048$	[3]	*	*
$f_+^{D \rightarrow K}(0)$	$0.747 \pm 0.011 \pm 0.034$	[3]	*	*

Table 2: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the “GS” column by a star; measurements or parameters that have systematic theoretical errors are marked in the “TH” column by a star. Upper part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

2 Lattice QCD averages

Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

2.1 Method of averaging

We collect the relevant calculations of the quantity that we are interested in: we take only unquenched results with 2 or 2+1 dynamical fermions, even those from proceedings without a companion article. For the calculations published before the end of November 2013, we have followed the classification of the Flavour Lattice Averaging Group [40] and kept only results with green squares. However, we stress that we perform our averages in a different manner from FLAG.

In these results, we separate the error estimates into a Gaussian part and a flat part that is treated à la Rfit. The Gaussian part collects the uncertainties from purely statistical origin, but also the systematics that can be controlled and treated in a similar way (e.g., interpolation or fitting in some cases). The remaining systematics constitute the Rfit error. If there are several sources of error in the Rfit category, we add them linearly ¹.

The Rfit model is simple but also very strict. It amounts to assuming that the theoretical uncertainty is rigorously constrained by a mathematical bound that is our only piece of information. If Rfit is taken *stricto sensu* and the individual likelihoods are combined in the usual way (by multiplication), the final uncertainty can be underestimated, in particular in the case of marginally compatible values.

We correct this effect by adopting the following averaging recipe. The central value is obtained by combining the whole likelihoods. Then we combine the Gaussian uncertainties by combining likelihoods restricted to their Gaussian part. Finally we assign to this combination the smallest of the individual Rfit uncertainties. The underlying idea is twofold:

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
- this best estimate should not be penalized by less precise methods (as it would happen be the case if one would take the dispersion of the individual central values as a guess of the combined theoretical uncertainty).

It should be stressed that the concept of a theoretical uncertainty is ill-defined, and the combination of them even more. Thus our approach is only one among the alternatives that can be found in the literature. In contrast to some of the latter, ours is algorithmic and can be reproduced.

3 Decay constants

3.1 Light mesons

f_K

¹keeping in mind that in many papers in the literature, this combination is done in quadrature and the splitting between different sources is not published.

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[41]	2	158.1	0.8	3.1
HPQCD07	[42]	2+1	157	0.6	3.3
MILC10	[43]	2+1	156.1	0.4	$^{+0.6}_{-0.9}$
Laiho11	[44]	2+1	156.8	1.0	3.3
RBC-UKQCD12	[45]	2+1	152.4	3.0	2.2
HPQCD13	[46]	2+1+1	155.4	0.2	0.6
ETMC13	[47]	2+1+1	155.6	1.6	2.2
Our average			155.2	0.2	0.6

f_K/f_π

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[41]	2	1.210	0.006	0.024
HPQCD/UKQCD07	[42]	2+1	1.189	0.002	0.014
MILC10	[43]	2+1	1.197	0.002	$^{+0.003}_{-0.007}$
BMW10	[48]	2+1	1.192	0.007	0.013
Laiho11	[44]	2+1	1.202	0.011	0.024
RBC-UKQCD12	[45]	2+1	1.1991	0.0116	0.0185
HPQCD13	[46]	2+1+1	1.1938	0.0015	0.0032
MILC13	[49]	2+1+1	1.1969	0.0026	0.0052
ETMC13	[47]	2+1+1	1.193	0.013	0.019
Our average			1.1942	0.0009	0.0032

Results have been corrected to express results in terms of the decay constants defined in QCD (electromagnetic corrections are applied at the level of the branching ratios).

3.2 Charmed mesons

f_{D_s}

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[41]	2	244	3	9
HPQCD10	[50]	2+1	248.0	1.4	4.5
FNAL-MILC11	[51]	2+1	260.1	8.9	16.2
FNAL-MILC12	[52]	2+1+1	246.4	0.5	5.6
ETMC13	[47]	2+1+1	242.1	7.6	4.6
Our average			245.3	0.5	4.5

f_{D_s}/f_D

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[41]	2	1.24	0.03	0.01
FNAL-MILC11	[51]	2+1	1.188	0.014	0.054
HPQCD12	[53]	2+1	1.187	0.004	0.023
FNAL-MILC12	[52]	2+1+1	1.175	0.016	0.018
ETMC13	[47]	2+1+1	1.199	0.017	0.023
Our average			1.201	0.004	0.010

3.3 Beauty mesons

f_{B_s}

Reference	Article	N_f	Mean	Stat	Syst
ETMC13	[54]	2	228	5	9
HPQCD11	[55]	2+1	225.0	2.9	5.4
FNAL-MILC11	[51]	2+1	242.0	5.1	21.2
HPQCD12	[56]	2+1	228.0	1.4	17.5
HPQCD13	[57]	2+1+1	224.0	2.5	7.2
Our average			225.6	1.1	5.4

f_{B_s}/f_B

Reference	Article	N_f	Mean	Stat	Syst
ETMC13	[54]	2	1.206	0.010	0.026
FNAL-MILC11	[51]	2+1	1.229	0.013	0.046
HPQCD12	[56]	2+1	1.188	0.012	0.025
HPQCD13	[57]	2+1	1.205	0.004	0.007
Our average			1.205	0.003	0.007

4 Semileptonic form factors

4.1 $K \rightarrow \pi \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
ETMC09	[58]	2	0.9560	0.0057	0.0118
MILC12	[59]	2+1	0.9667	0.0023	0.0077
RBC-UKQCD13	[60]	2+1	0.9670	0.0020	$^{+0.024}_{-0.066}$
Our average			0.9641	0.0015	0.0045

Combining with $|V_{us}|f_+(0) = 0.2166(5)$ from ref.[61], we get $|V_{us}| = 0.2247 \pm 0.0006 \pm 0.0011$.

4.2 $D \rightarrow \pi \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
ETMC11	[62]	2	0.65	0.06	0.12
HPQCD11	[63]	2+1	0.666	0.021	0.048
Our average			0.666	0.020	0.048

4.3 $D \rightarrow K\ell\nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
ETMC11	[62]	2	0.76	0.05	0.11
HPQCD10	[64]	2+1	0.747	0.011	0.034
Our average			0.747	0.011	0.034

5 Meson mixing

5.1 Kaon mixing

$B_K^{\text{MS}}(2\text{GeV})$

Reference	Article	N_f	Mean	Stat	Syst
ETMC 10	[65]	2	0.532	0.019	0.026
Laiho11	[44]	2+1	0.5572	0.0028	0.0257
BMW11	[66]	2+1	0.5644	0.0059	0.0100
RBC-UKQCD12	[45]	2+1	0.549	0.010	0.030
SWME14	[67]	2+1	0.5388	0.0034	0.0442
Our average for $B_K^{\text{MS}}(2\text{GeV})$			0.5562	0.0020	0.0100
Our average for \hat{B}_K			0.7615	0.0027	0.0137

5.2 $B_{d,s}$ mixing

\hat{B}_{B_s}

Reference	Article	N_f	Mean	Stat	Syst
ETMC13	[54]	2	1.32	0.04	0.03
HPQCD09	[68]	2+1	1.326	0.018	0.040
Our average			1.32	0.017	0.030

Ref. [68] provide only f_{B_s} and $f_{B_s}\sqrt{\hat{B}_{B_s}}$, and we assumed that the systematics were completely correlated to extract \hat{B}_{B_s} .

$\hat{B}_{B_s}/\hat{B}_{B_d}$

Reference	Article	N_f	Mean	Stat	Syst
ETMC13	[54]	2	1.007	0.015	0.014
HPQCD09	[68]	2+1	1.053	0.025	0.023
FNAL/MILC12	[69]	2+1	1.064	0.076	0.193
Our average			1.023	0.013	0.014

Refs. [68] and [69] provide only ξ and f_{B_s}/f_{B_d} . For Refs. [68], we have extracted $\hat{B}_{B_s}/\hat{B}_{B_d}$ in both cases assuming a total correlation in the systematics of ξ and $\hat{B}_{B_s}/\hat{B}_{B_d}$. For Ref. [69], we have considered all uncertainties as uncorrelated, as the studies of the decay constants and the bag parameters have been performed in different settings, with different categories of systematics.

6 Results

Observable	central \pm CL \equiv 1 σ	\pm CL \equiv 2 σ	\pm CL \equiv 3 σ
A	$0.813^{+0.015}_{-0.027}$	$0.813^{+0.021}_{-0.035}$	$0.813^{+0.029}_{-0.040}$
λ	$0.22551^{+0.00068}_{-0.00035}$	$0.22551^{+0.00093}_{-0.00069}$	$0.2255^{+0.0010}_{-0.0010}$
$\bar{\rho}$	$0.1489^{+0.0158}_{-0.0084}$	$0.149^{+0.033}_{-0.016}$	$0.149^{+0.043}_{-0.024}$
$\bar{\eta}$	$0.342^{+0.013}_{-0.011}$	$0.342^{+0.024}_{-0.024}$	$0.342^{+0.036}_{-0.036}$
J [10^{-5}]	$2.97^{+0.18}_{-0.20}$	$2.97^{+0.30}_{-0.25}$	$2.97^{+0.41}_{-0.30}$
$\sin 2\alpha$	$-0.058^{+0.055}_{-0.092}$	$-0.058^{+0.099}_{-0.192}$	$-0.06^{+0.14}_{-0.25}$
$\sin 2\alpha$ (!)	$-0.12^{+0.10}_{-0.11}$	$-0.12^{+0.15}_{-0.17}$	$-0.12^{+0.19}_{-0.22}$
$\sin 2\beta$	$0.692^{+0.020}_{-0.018}$	$0.692^{+0.039}_{-0.036}$	$0.692^{+0.057}_{-0.051}$
$\sin 2\beta$ (!)	$0.774^{+0.017}_{-0.036}$	$0.774^{+0.034}_{-0.090}$	$0.774^{+0.050}_{-0.139}$
α [$^\circ$]	$91.7^{+2.6}_{-1.6}$	$91.7^{+5.6}_{-2.8}$	$91.7^{+7.3}_{-4.1}$
α [$^\circ$] (!)	$93.6^{+3.2}_{-2.9}$	$93.6^{+4.8}_{-4.2}$	$93.6^{+6.4}_{-5.6}$
α [$^\circ$] (dir. meas.)	$85.4^{+4.0}_{-3.9}$	$85.4^{+10.4}_{-7.8} 0.2^{+9.6}_{-8.8}$	$85^{+17}_{-12} 0^{+15}_{-15}$
β [$^\circ$]	$21.88^{+0.81}_{-0.71}$	$21.9^{+1.6}_{-1.4}$	$21.9^{+2.4}_{-2.0}$
β [$^\circ$] (!)	$25.38^{+0.80}_{-1.57}$	$25.4^{+1.6}_{-3.8}$	$25.4^{+2.4}_{-5.7}$
β [$^\circ$] (dir. meas.)	$21.50^{+0.75}_{-0.74}$	$21.5^{+1.5}_{-1.5}$	$21.5^{+2.3}_{-2.2}$
γ [$^\circ$]	$66.5^{+1.3}_{-2.5}$	$66.5^{+2.4}_{-5.1}$	$66.5^{+3.4}_{-6.4}$
γ [$^\circ$] (!)	$66.4^{+1.2}_{-3.3}$	$66.4^{+2.3}_{-5.4}$	$66.4^{+3.4}_{-6.6}$
γ [$^\circ$] (dir. meas.)	$70.0^{+7.7}_{-9.0}$	70^{+15}_{-18}	$70^{+22}_{-27} 42.16^{+0.29}_{-0.15}$
R_u	$0.373^{+0.013}_{-0.011}$	$0.373^{+0.025}_{-0.022}$	$0.373^{+0.038}_{-0.032}$
R_t	$0.9171^{+0.0082}_{-0.0166}$	$0.917^{+0.015}_{-0.033}$	$0.917^{+0.022}_{-0.042}$
$\bar{\rho}_s$	$-0.00797^{+0.00046}_{-0.00085}$	$-0.00797^{+0.00089}_{-0.00183}$	$-0.0080^{+0.0013}_{-0.0023}$
$\bar{\eta}_s$	$-0.01832^{+0.00060}_{-0.00068}$	$-0.0183^{+0.0013}_{-0.0013}$	$-0.0183^{+0.0019}_{-0.0020}$
$\beta_s \equiv -\arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right)$ [rad]	$0.01817^{+0.00068}_{-0.00060}$	$0.0182^{+0.0013}_{-0.0013}$	$0.0182^{+0.0019}_{-0.0019}$
$\sin 2\beta_s$	$0.0363^{+0.0014}_{-0.0012}$	$0.0363^{+0.0026}_{-0.0026}$	$0.0363^{+0.0039}_{-0.0038}$

Observable	central \pm CL \equiv 1σ	\pm CL \equiv 2σ	\pm CL \equiv 3σ
$ V_{ud} $	$0.974235^{+0.000080}_{-0.000158}$	$0.97424^{+0.00016}_{-0.00022}$	$0.97424^{+0.00024}_{-0.00024}$
$ V_{us} $	$0.22551^{+0.00068}_{-0.00034}$	$0.22551^{+0.00094}_{-0.00069}$	$0.2255^{+0.0010}_{-0.0010}$
$ V_{ub} $	$0.00357^{+0.00016}_{-0.00015}$	$0.00357^{+0.00029}_{-0.00024}$	$0.00357^{+0.00041}_{-0.00031}$
$ V_{cd} $	$0.22537^{+0.00068}_{-0.00035}$	$0.22537^{+0.00093}_{-0.00069}$	$0.2254^{+0.0010}_{-0.0010}$
$ V_{cs} $	$0.973395^{+0.000095}_{-0.000176}$	$0.97340^{+0.00019}_{-0.00024}$	$0.97340^{+0.00028}_{-0.00027}$
$ V_{cb} $	$0.04136^{+0.00071}_{-0.00128}$	$0.0414^{+0.0010}_{-0.0016}$	$0.0414^{+0.0014}_{-0.0018}$
$ V_{td} $	$0.00855^{+0.00018}_{-0.00030}$	$0.00855^{+0.00028}_{-0.00048}$	$0.00855^{+0.00038}_{-0.00057}$
$ V_{ts} $	$0.04062^{+0.00070}_{-0.00125}$	$0.0406^{+0.0010}_{-0.0016}$	$0.0406^{+0.0014}_{-0.0018}$
$ V_{tb} $	$0.999138^{+0.000052}_{-0.000030}$	$0.999138^{+0.000065}_{-0.000044}$	$0.999138^{+0.000073}_{-0.000058}$
$ V_{ud} $ (!)	$0.974236^{+0.000080}_{-0.000179}$	$0.97424^{+0.00016}_{-0.00032}$	$0.97424^{+0.00024}_{-0.00045}$
$ V_{us} $ (!)	$0.224488^{+0.001909}_{-0.000066}$	$0.22449^{+0.00202}_{-0.00013}$	$0.22449^{+0.00209}_{-0.00020}$
$ V_{ub} $ (!)	$0.003435^{+0.000250}_{-0.000084}$	$0.00343^{+0.00038}_{-0.00017}$	$0.00343^{+0.00051}_{-0.00025}$
$ V_{cb} $ (!)	$0.0414^{+0.0024}_{-0.0014}$	$0.0414^{+0.0028}_{-0.0018}$	$0.0414^{+0.0032}_{-0.0021}$

Observable	central \pm CL \equiv 1 σ	\pm CL \equiv 2 σ	\pm CL \equiv 3 σ
Δm_d [ps $^{-1}$] (!)	0.576 $^{+0.041}_{-0.046}$	0.576 $^{+0.074}_{-0.087}$	0.58 $^{+0.11}_{-0.13}$
Δm_s [ps $^{-1}$] (!)	16.3 $^{+1.2}_{-1.2}$	16.3 $^{+2.8}_{-1.6}$	16.3 $^{+3.8}_{-2.0}$
$ \epsilon_K $ [10 $^{-3}$] (!)	2.08 $^{+0.58}_{-0.57}$	2.08 $^{+0.70}_{-0.67}$	2.08 $^{+0.82}_{-0.73}$
m_t [GeV/c 2] (!)	160.2 $^{+12.8}_{-5.0}$	160.2 $^{+25.7}_{-7.2}$	160.2 $^{+29.2}_{-9.4}$
B_K (lattice value not in the fit)	0.84 $^{+0.26}_{-0.19}$	0.84 $^{+0.34}_{-0.22}$	0.84 $^{+0.40}_{-0.25}$
f_{B_s}/f_{B_d} (lattice value not in the fit)	1.258 $^{+0.046}_{-0.036}$	1.258 $^{+0.084}_{-0.096}$	1.26 $^{+0.12}_{-0.15}$
f_{B_s} (lattice value not in the fit)	0.2372 $^{+0.0024}_{-0.0169}$	0.2372 $^{+0.0047}_{-0.0198}$	0.2372 $^{+0.0070}_{-0.0221}$
B_{B_s}/B_{B_d} (lattice value not in the fit)	1.163 $^{+0.064}_{-0.078}$	1.16 $^{+0.13}_{-0.16}$	1.16 $^{+0.20}_{-0.26}$
B_{B_s} (lattice value not in the fit)	1.250 $^{+0.132}_{-0.035}$	1.250 $^{+0.282}_{-0.057}$	1.250 $^{+0.314}_{-0.079}$
$\mathcal{B}(B^+ \rightarrow \tau\nu)$ [10 $^{-4}$]	0.817 $^{+0.059}_{-0.075}$	0.82 $^{+0.13}_{-0.13}$	0.82 $^{+0.19}_{-0.17}$
$\mathcal{B}(B^+ \rightarrow \tau\nu)$ [10 $^{-4}$] (!)	0.753 $^{+0.102}_{-0.052}$	0.753 $^{+0.167}_{-0.092}$	0.75 $^{+0.23}_{-0.13}$
$\mathcal{B}(B^+ \rightarrow \mu\nu)$ [10 $^{-6}$]	0.367 $^{+0.026}_{-0.034}$	0.367 $^{+0.057}_{-0.060}$	0.367 $^{+0.086}_{-0.078}$
$\mathcal{B}(B^+ \rightarrow e\nu)$ [10 $^{-11}$]	0.859 $^{+0.062}_{-0.079}$	0.86 $^{+0.13}_{-0.14}$	0.86 $^{+0.20}_{-0.18}$
$\mathcal{B}(B_d \rightarrow e^+e^-)$ [10 $^{-15}$]	2.55 $^{+0.15}_{-0.23}$	2.55 $^{+0.20}_{-0.32}$	2.55 $^{+0.26}_{-0.38}$
$\mathcal{B}(B_d \rightarrow \mu^+\mu^-)$ [10 $^{-11}$]	10.87 $^{+0.63}_{-0.97}$	10.87 $^{+0.87}_{-1.39}$	10.9 $^{+1.1}_{-1.6}$
$\mathcal{B}(B_s \rightarrow e^+e^-)$ [10 $^{-14}$]	8.55 $^{+0.43}_{-0.71}$	8.55 $^{+0.56}_{-0.83}$	8.55 $^{+0.69}_{-0.94}$
$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ [10 $^{-9}$]	3.65 $^{+0.18}_{-0.30}$	3.65 $^{+0.24}_{-0.35}$	3.65 $^{+0.29}_{-0.40}$
$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ [10 $^{-9}$] (!)	3.65 $^{+0.18}_{-0.30}$	3.65 $^{+0.24}_{-0.35}$	3.65 $^{+0.29}_{-0.40}$
$\mathcal{B}(D_s \rightarrow \tau^+\nu)$ (!)	0.05187 $^{+0.00021}_{-0.00126}$	0.05187 $^{+0.00042}_{-0.00312}$	0.05187 $^{+0.00062}_{-0.00403}$
$\mathcal{B}(D_s \rightarrow \mu^+\nu)$ [10 $^{-2}$] (!)	0.5313 $^{+0.0021}_{-0.0105}$	0.5313 $^{+0.0043}_{-0.0276}$	0.5313 $^{+0.0064}_{-0.0405}$
$\mathcal{B}(D \rightarrow \mu^+\nu)$ [10 $^{-3}$] (!)	0.390 $^{+0.011}_{-0.011}$	0.390 $^{+0.015}_{-0.019}$	0.390 $^{+0.018}_{-0.030}$
$\mathcal{B}(K \rightarrow \mu^+\nu)$ (!)	0.6373 $^{+0.0023}_{-0.0030}$	0.6373 $^{+0.0043}_{-0.0061}$	0.6373 $^{+0.0061}_{-0.0091}$
$\mathcal{B}(K \rightarrow e^+\nu)$ [10 $^{-4}$] (!)	0.15692 $^{+0.00047}_{-0.00047}$	0.15692 $^{+0.00092}_{-0.00093}$	0.1569 $^{+0.0013}_{-0.0013}$
$\mathcal{B}(\tau^+ \rightarrow K\nu)$ [10 $^{-2}$] (!)	0.7179 $^{+0.0015}_{-0.0015}$	0.7179 $^{+0.0031}_{-0.0031}$	0.7179 $^{+0.0046}_{-0.0046}$

(!) means that the measurement was not included in the fit

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