UPDATED RESULTS ON THE CKM MATRIX

Including results presented up to
Summer 19

P r e l i m i n a r y

Dec 15th, 2019

The CKMfitter Group

Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach, using the Rfit model to treat theoretical uncertainties. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

CP VIOLATION AND THE CKM MATRIX:
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES
By CKMfitter Group
Eur. Phys. J. **C41**, 1-131, 2005 [hep-ph/0406184]

The CKMfitter Group

J. Charles[a], O. Deschamps[b], S. Descotes-Genon[c], S. Monteil[b], J. Orloff[b],
W. Qian[d], V. Tisserand[b], K. Trabelsi[e], P. Urquijo[f], L. Vale Silva[g]

[a] *Aix Marseille Université, Université de Toulon, CNRS, CPT UMR 7332, 13288, Marseille, Franc*
e-mail: charles@cpt.univ-mrs.fr

[b] *Laboratoire de Physique Corpusculaire de Clermont-Ferrand*
Université Blaise Pascal, 24 Avenue des Landais F-63177 Aubière Cedex, France
(UMR 6533 du CNRS-IN2P3 associée à l'Université Blaise Pascal)
e-mail: odescham@in2p3.fr, monteil@in2p3.fr, orloff@in2p3.fr,Vincent.Tisserand@clermont.in2p3.fr

[c] *Laboratoire de Physique Théorique*
UMR8627, CNRS, Univ. Paris-Sud, Université Paris Saclay, F-91405 Orsay Cedex, France
e-mail: Sebastien.Descotes-Genon@th.u-psud.fr

[d] *University of Chinese Academy of Sciences*
(associated to Center for High Energy Physics, Tsinghua University), Beijing, China
e-mail: wenbin.qian@cern.ch

[e] *Laboratoire de l'Accélérateur Linéaire, CNRS/IN2P3, Univ. Paris-Sud, Université Paris-Saclay*
F-91405 Orsay Cedex, France
e-mail: trabelsi@lal.in2p3.fr

[f] *School of Physics, University of Melbourne, Victoria 3010, Australia*
e-mail: Phillip.Urquijo@cern.ch

[g] *University of Sussex, Department of Physics and Astronomy, Falmer, Brighton BN1 9QH, UK*
e-mail: L.H.Vale-Silva@sussex.ac.uk

1 Inputs

2 Lattice QCD averages

Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

3 Method of averaging

We collect the relevant calculations of the quantity that we are interested in: we take results with 2, 2+1 or 2+1+1 dynamical fermions, even those from proceedings without a companion article. For data prior to the publication of the latest FLAG review [42], we followed their prescriptions to discard results of insufficient quality.

In these results, we separate the error estimates into a Gaussian part and a flat part that is treated à la Rfit. The Gaussian part collects the uncertainties from purely statistical origin, but also the systematics that can be controlled and treated in a similar way (e.g., interpolation or fitting in some cases). The remaining systematics constitute the Rfit error. If there are several sources of error in the Rfit category, we add them linearly [1].

The Rfit model is simple but also very strict. It amounts to assuming that the theoretical uncertainty is rigorously constrained by a mathematical bound that is our only piece of information. If Rfit is taken stricto sensu and the individual likelihoods are combined in the usual way (by multiplication), the final uncertainty can be underestimated, in particular in the case of marginally compatible values.

We correct this effect by adopting the following averaging recipe. The central value is obtained by combining the whole likelihoods. Then we combine the Gaussian uncertainties by combining likelihoods restricted to their Gaussian part. Finally we assign to this combination the smallest of the individual Rfit uncertainties. The underlying idea is twofold:

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates

- this best estimate should not be penalized by less precise methods (as it would happen be the case if one would take the dispersion of the individual central values as a guess of the combined theoretical uncertainty).

It should be stressed that the concept of a theoretical uncertainty is ill-defined, and the combination of them even more. Thus our approach is only one among the alternatives that can be found in the literature. In contrast to some of the latter, ours is algorithmic and can be reproduced.

---

[1]keeping in mind that in many papers in the literature, this combination is done in quadrature and the splitting between different sources is not published.

Parameter	Value ± Error(s)	Reference	GS	TH	Tree-level fit
$|V_{ud}|$ (nuclei)	$0.97418 \pm 0 \pm 0.00021$	[1]	-	★	★
$|V_{us}|f_+^{K\to\pi}(0)$	0.2165 ± 0.0004	[2]	★	★	★
$|V_{cd}|$ (νN)	0.230 ± 0.011	[2]	★	-	★
$|V_{cs}|$ ($W\to c\bar{s}$)	$0.94^{+0.32}_{-0.26} \pm 0.13$	[2]	★	★	★
$|V_{ub}|$ (semileptonic)	$(3.91 \pm 0.08 \pm 0.21)\times 10^{-3}$	[3,5]	★	★	★
$|V_{cb}|$ (semileptonic)	$(41.1 \pm 0.3 \pm 0.5)\times 10^{-3}$	[3,6]	★	★	★
$\mathcal{B}(\Lambda_p \to p\mu^-\overline{\nu}_\mu)_{q^2>15}/\mathcal{B}(\Lambda_p \to \Lambda_c\mu^-\overline{\nu}_\mu)_{q^2>7}$	$(0.947 \pm 0.081)\times 10^{-2}$	[7,8]	★	-	★
$\mathcal{B}(B^- \to \tau^-\overline{\nu}_\tau)$	$(1.09 \pm 0.24)\times 10^{-4}$	[3,9]	★	-	★
$\mathcal{B}(D_s^- \to \mu^-\overline{\nu}_\mu)$	$(5.51 \pm 0.16)\times 10^{-3}$	[3]	★	-	★
$\mathcal{B}(D_s^- \to \tau^-\overline{\nu}_\tau)$	$(5.52 \pm 0.24)\times 10^{-2}$	[3]	★	-	★
$\mathcal{B}(D^- \to \mu^-\overline{\nu}_\mu)$	$(3.77 \pm 0.18)\times 10^{-4}$	[3]	★	★	★
$\mathcal{B}(D^- \to \tau^-\overline{\nu}_\tau)$	$(1.20 \pm 0.27)\times 10^{-3}$	[3]	★	-	★
$\mathcal{B}(K^- \to e^-\overline{\nu}_e)$	$(1.582 \pm 0.007)\times 10^{-5}$	[2]	★	-	★
$\mathcal{B}(K^- \to \mu^-\overline{\nu}_\mu)$	0.6356 ± 0.0011	[2]	★	-	★
$\mathcal{B}(\tau^- \to K^-\overline{\nu}_\tau)$	$(0.6986 \pm 0.0085)\times 10^{-2}$	[3]	★	-	★
$\mathcal{B}(K^- \to \mu^-\overline{\nu}_\mu)/\mathcal{B}(\pi^- \to \mu^-\overline{\nu}_\mu)$	1.3367 ± 0.0029	[2]	★	-	★
$\mathcal{B}(\tau^- \to K^-\overline{\nu}_\tau)/\mathcal{B}(\tau^- \to \pi^-\overline{\nu}_\tau)$	$(6.438 \pm 0.094)\times 10^{-2}$	[3]	★	-	★
$\mathcal{B}(B_s \to \mu\mu)$	$(2.8^{+0.7}_{-0.6})\times 10^{-9}$	[11]	★	-	-
$|V_{cd}|f_+^{D\to\pi}(0)$	0.1426 ± 0.0018	[3]	★	-	★
$|V_{cs}|f_+^{D\to K}(0)$	0.7180 ± 0.0033	[3]	★	-	★
$|\varepsilon_K|$	$(2.228 \pm 0.011)\times 10^{-3}$	[2]	★	-	-
Δm_d	$(0.5065 \pm 0.0019)\ \mathrm{ps}^{-1}$	[3]	★	-	-
Δm_s	$(17.757 \pm 0.021)\ \mathrm{ps}^{-1}$	[3]	★	-	-
$\sin(2\beta)_{[c\bar{c}]}$	0.699 ± 0.017	[3]	★	-	-
$(\phi_s)_{[b\to c\bar{s}s]}$	-0.055 ± 0.021	[3]	★	-	-
$S_{\pi\pi}^{+-}, C_{\pi\pi}^{+-}, C_{\pi\pi}^{00}, \mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis	[12–20]	★	-	-
$S_{\rho\rho,L}^{+-}, C_{\rho\rho,L}^{+-}, S_{\rho\rho}^{00}, C_{\rho\rho}^{00}, \mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[21–27]	★	-	-
$B^0 \to (\rho\pi)^0 \to 3\pi$	Time-dependent Dalitz analysis	[28,29]	★	-	-
$B^- \to D^{(*)}K^{(*)-}$	Inputs to GLW analysis	[30,32]	★	-	★
$B^- \to D^{(*)}K^{(*)-}$	Inputs to ADS analysis	[31,32]	★	-	★
$B^- \to D^{(*)}K^{(*)-}$	GGSZ Dalitz analysis	[33,34]	★	-	★

Table 1: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. Two columns indicate Rfit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the "GS" column by a star; measurements or parameters that have systematic theoretical errors are marked in the "TH" column by a star. The last column indicates by a star which observables are included in the tree-level fit observable. Upper part: experimental determinations of the CKM matrix elements. Lower part: CP-violation and mixing observables.*

Parameter	Value $\pm$ Error(s)	Reference	Errors GS	TH
$\overline{m}_c(m_c)$	$(1.2982 \pm 0.0013 \pm 0.0120)\,\mathrm{GeV}$	[4]	$\star$	$\star$
$\overline{m}_t(m_t)$	$(165.26 \pm 0.11 \pm 0.30)\,\mathrm{GeV}$	[35]	$\star$	$\star$
$\alpha_s(m_Z)$	$0.1181 \pm 0 \pm 0.0011$	[2]	-	$\star$
B_K	$0.7567 \pm 0.0021 \pm 0.0123$	[4]	$\star$	$\star$
κ_ϵ	$0.940 \pm 0.013 \pm 0.023$	[37, 38]	$\star$	$\star$
η_{ut}	$0.402 \pm 0 \pm 0.007$	[39]	-	$\star$
η_{tt}	$0.55 \pm 0 \pm 0.024$	[39]	-	$\star$
$\eta_B(\overline{\mathrm{MS}})$	$0.5510 \pm 0 \pm 0.0022$	[40, 41]	-	$\star$
f_{B_s}	$(228.8 \pm 0.7 \pm 1.9)\,\mathrm{MeV}$	[4]	$\star$	$\star$
B_s	$1.327 \pm 0.016 \pm 0.030$	[4]	$\star$	$\star$
f_{B_s}/f_{B_d}	$1.205 \pm 0.004 \pm 0.006$	[4]	$\star$	$\star$
B_s/B_d	$1.007 \pm 0.013 \pm 0.014$	[4]	$\star$	$\star$
f_K	$(155.6 \pm 0.2 \pm 0.6)\,\mathrm{MeV}$	[4]	$\star$	$\star$
f_K/f_π	$1.1973 \pm 0.0008 \pm 0.0014$	[4]	$\star$	$\star$
f_{D_s}	$(249.2 \pm 0.3 \pm 0.7)\,\mathrm{MeV}$	[4]	$\star$	$\star$
f_{D_s}/f_D	$1.1782 \pm 0.0006 \pm 0.0033$	[4]	$\star$	$\star$
$f_+^{K\to\pi}(0)$	$0.9681 \pm 0.0014 \pm 0.0022$	[4]	$\star$	$\star$
$f_+^{D\to\pi}(0)$	$0.621 \pm 0.018 \pm 0.012$	[4]	$\star$	$\star$
$f_+^{D\to K}(0)$	$0.765 \pm 0.010 \pm 0.012$	[4]	$\star$	$\star$
$\zeta(\Lambda_p \to p\mu^-\overline{\nu}_\mu)_{q^2>15}/\zeta(\Lambda_p \to \Lambda_c\mu^-\overline{\nu}_\mu)_{q^2>7}$	$1.471 \pm 0.096 \pm 0.290$	[4]	$\star$	$\star$

Table 2: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate R*fit *treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the "GS" column by a star; measurements or parameters that have systematic theoretical errors are marked in the "TH" column by a star.* <u>Upper part</u>: *parameters used in SM predictions that are obtained from experiment.* <u>Lower part</u>: *parameters of the SM predictions obtained from theory.*

4 Decay constants

4.1 Light mesons

f_K

Reference	Article	N_f	Mean	Stat	Syst
ETM09	[43]	2	158.1	0.8	3.1
HPQCD07	[44]	2+1	157	0.6	3.3
MILC10	[45]	2+1	156.1	0.4	$^{+0.6}_{-0.9}$
HPQCD13	[46]	2+1+1	155.4	0.2	0.6
ETM14	[47]	2+1+1	155.0	1.4	2.0
Our average			155.57	0.17	0.57

f_K/f_π

Reference	Article	N_f	Mean	Stat	Syst
ETM09	[43]	2	1.210	0.006	0.024
HPQCD/UKQCD07	[44]	2+1	1.189	0.002	0.014
BMW10	[48]	2+1	1.192	0.007	0.013
MILC10	[45]	2+1	1.197	0.002	$^{+0.003}_{-0.007}$
BMW16	[49]	2+1	1.182	0.010	0.026
QCDSF/UKQCD16	[50]	2+1	1.192	0.010	0.026
HPQCD13	[46]	2+1+1	1.1948	0.0015	0.0039
ETM14	[47]	2+1+1	1.188	0.011	0.020
FNAL/MILC17	[51]	2+1+1	1.198	0.0012	$^{+0.0007}_{-0.0021}$
Our average			1.1973	0.0008	0.0014

The results have been corrected to yield the decay constants defined in QCD (electromagnetic corrections are applied at the level of the branching ratios).

4.2 Charmed mesons

f_{D_s}

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[52]	2	250.0	5	5
HPQCD10	[53]	2+1	248.0	1.4	4.5
FNAL-MILC11	[54]	2+1	260.1	8.9	16.2
ChiQCD14	[55]	2+1	254.0	2.2	10.2
RBC-UKQCD17	[56]	2+1	246.4	1.9	$^{+2.5}_{-3.7}$
ETM14	[47]	2+1+1	247.2	3.9	2.2
FNAL/MILC17	[51]	2+1+1	249.9	0.28	0.65
Our average			249.24	0.27	0.65

f_{D_s}/f_D

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[52]	2	1.201	0.007	0.020
FNAL-MILC11	[54]	2+1	1.188	0.014	0.054
HPQCD12	[57]	2+1	1.187	0.004	0.023
RBC-UKQCD17	[56]	2+1	1.1667	0.0077	$^{+0.0090}_{-0.0069}$
ETM14	[47]	2+1+1	1.192	0.019	0.017
FNAL/MILC17	[51]	2+1+1	1.1749	0.0006	0.0033
Our average			1.1782	0.0006	0.0033

4.3 Beauty mesons

f_{B_s}

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[52]	2	228	5	9
ALPHA14	[58]	2	224	14	2
HPQCD11	[59]	2+1	225.0	2.9	5.4
FNAL-MILC11	[54]	2+1	242.0	5.1	21.2
RBC-UKQCD14	[60]	2+1	235.4	5.2	28.1
HPQCD13	[61]	2+1+1	224.0	2.5	7.2
ETM16	[62]	2+1+1	229	3.9	4.8
HPQCD17	[63]	2+1+1	236	2.6	10.0
FNAL/MILC17	[51]	2+1+1	230.7	0.8	1.9
Our average			228.82	0.69	1.87

Ref. [64] is not included due to the significant correlation with HPQCD13, as discussed in Ref. [65]. The same argument holds for Ref. [66] compared to RBC-UKQCD14 (with the additional issue of the uncertainty from the infinite quark limit). Only (subdominant) statistical uncertainties are correlated for HPQCD13 and HPQCD18, and we keep both determinations.

f_{B_s}/f_B

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[52]	2	1.206	0.010	0.026
ALPHA14	[58]	2	1.203	0.062	0.019
FNAL-MILC11	[54]	2+1	1.229	0.013	0.046
RBC-UKQCD14	[60]	2+1	1.223	0.013	0.106
HPQCD13	[61]	2+1+1	1.205	0.004	0.007
ETM16	[62]	2+1+1	1.184	0.018	0.028
HPQCD17	[63]	2+1+1	1.207	0.003	0.006
FNAL/MILC17	[51]	2+1+1	1.2180	0.0033	0.0058
Our average			1.2050	0.0034	0.0063

Ref. [64] is not included due to the significant correlation with HPQCD13, as discussed in Ref. [65]. The same argument for Ref. [66] compared to RBC-UKQCD14 (with the additional issue of the uncertainty from the infinite quark limit). Only (subdominant) statistical uncertainties are correlated for HPQCD13 and HPQCD18, and we keep both determinations.

5 Semileptonic form factors

5.1 $K \to \pi \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
ETM09	[67]	2	0.9560	0.0057	0.0118
MILC12	[68]	2+1	0.9667	0.0023	0.0055
RBC-UKQCD15	[69]	2+1	0.9670	0.0034	0.0019
MILC13	[70]	2+1+1	0.9704	0.0024	0.0022
ETM16	[71]	2+1+1	0.9697	0.0045	0.0036
Our average			0.9681	0.0014	0.0022

5.2 $D \to \pi \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
HPQCD11	[72]	2+1	0.666	0.021	0.048
ETM17	[73]	2+1+1	0.612	0.035	0.012
Our average			0.621	0.018	0.012

5.3 $D \to K \ell \nu$

$f_+(0)$

Reference	Article	N_f	Mean	Stat	Syst
HPQCD10	[74]	2+1	0.747	0.011	0.034
ETM17	[73]	2+1+1	0.765	0.029	0.012
Our average			0.765	0.010	0.012

5.4 $\Lambda_b \to p \mu^- \bar{\nu}_\mu$ and $\Lambda_b \to \Lambda_c \bar{\nu}_\mu$

$\zeta(\Lambda_b \to p \mu^- \bar{\nu}_\mu)_{q^2>15} / \zeta(\Lambda_b \to \Lambda_c \mu^- \bar{\nu}_\mu)_{q^2>7}$

Reference	Article	N_f	Mean	Stat	Syst
DLM15	[75]	2+1	1.471	0.096	0.290
Our average			1.471	0.096	0.290

6 Meson mixing

6.1 Kaon mixing

$B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$

Reference	Article	N_f	Mean	Stat	Syst
ETM12	[76]	2	0.531	0.016	0.009
LVdW11	[77]	2+1	0.5572	0.0028	0.0257
BMW11	[78]	2+1	0.5644	0.0059	0.0100
RBC-UKQCD14	[79]	2+1	0.5477	0.002	0.011
SWME15	[80]	2+1	0.537	0.004	0.041
ETM15	[81]	4	0.5237	0.0131	0.0351
Our average for $B_K^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$			0.5527	0.0015	0.0090
Our average for $\hat{B}_K$			0.7567	0.0021	0.0123

6.2 $B_{d,s}$ mixing

$\hat{B}_{B_s}$

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[52]	2	1.32	0.04	0.03
HPQCD09	[82]	2+1	1.326	0.018	0.040
RBC-UKQCD14	[66]	2+1	1.22	0.06	0.32
FNAL-MILC16	[83]	2+1	1.443	0.078	0.138
Our average			1.327	0.016	0.030

Ref. [82] provide only f_{B_s} and $f_{B_s}\sqrt{\hat{B}_{B_s}}$, and we assumed that the systematics were completely correlated to extract $\hat{B}_{B_s}$.

$\hat{B}_{B_s}/\hat{B}_{B_d}$

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[52]	2	1.007	0.015	0.014
RBC-UKQCD14	[66]	2+1	1.028	0.060	0.100
FNAL-MILC16	[83]	2+1	1.033	0.029	0.048
Our average			1.007	0.013	0.014

7 Quark masses

$\bar{m}_c(\bar{m}_c)$

Reference	Article	N_f	Mean	Stat	Syst
HPQCD10	[84]	2+1	1.273	0.0022	0.0120
ChiQCD14	[55]	2+1	1.304	0.0050	0.0449
ETM14A	[85]	2+1+1	1.3478	0.0027	0.0195
HQPCD14A	[86]	2+1+1	1.2715	0.0030	0.0225
HQPCD18	[87]	2+1+1	1.2757	0.0036	0.0208
FNAL/MILC/TUMQCD18	[88]	2+1+1	1.273	0.004	0.012
Our average			1.2982	0.0013	0.012

8 Results of the CKM global fit

8.1 Global fit

Observable	central $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
A	$0.8235\,^{+0.0056}_{-0.0145}$	$0.823\,^{+0.011}_{-0.023}$	$0.823\,^{+0.015}_{-0.029}$
λ	$0.224837\,^{+0.000251}_{-0.000060}$	$0.22484\,^{+0.00060}_{-0.00012}$	$0.22484\,^{+0.00097}_{-0.00018}$
$\bar{\rho}$	$0.1569\,^{+0.0102}_{-0.0061}$	$0.157\,^{+0.027}_{-0.012}$	$0.157\,^{+0.036}_{-0.019}$
$\bar{\eta}$	$0.3499\,^{+0.0079}_{-0.0065}$	$0.350\,^{+0.018}_{-0.016}$	$0.350\,^{+0.027}_{-0.023}$
J $[10^{-5}]$	$3.060\,^{+0.071}_{-0.079}$	$3.06\,^{+0.17}_{-0.13}$	$3.06\,^{+0.27}_{-0.18}$
$\sin 2\alpha$	$-0.057\,^{+0.038}_{-0.052}$	$-0.057\,^{+0.076}_{-0.146}$	$-0.06\,^{+0.11}_{-0.19}$
$\sin 2\alpha$ (meas. not in the fit)	$-0.069\,^{+0.040}_{-0.064}$	$-0.069\,^{+0.080}_{-0.153}$	$-0.07\,^{+0.12}_{-0.19}$
$\sin 2\beta$	$0.7083\,^{+0.0127}_{-0.0098}$	$0.708\,^{+0.027}_{-0.021}$	$0.708\,^{+0.042}_{-0.031}$
$\sin 2\beta$ (meas. not in the fit)	$0.731\,^{+0.029}_{-0.027}$	$0.731\,^{+0.061}_{-0.042}$	$0.731\,^{+0.078}_{-0.056}$
α [°]	$91.7\,^{+1.7}_{-1.1}$	$91.7\,^{+4.1}_{-2.3}$	$91.7\,^{+5.4}_{-3.3}$
α [°] (meas. not in the fit)	$91.8\,^{+2.7}_{-1.0}$	$91.8\,^{+4.5}_{-2.2}$	$91.8\,^{+5.7}_{-3.2}$
α [°] (dir. meas.)	$86.4\,^{+4.5}_{-4.3}|| - 1.8\,^{+4.3}_{-5.1}$	$86.4\,^{+12.3}_{-8.0}|| - 1.8\,^{+11.0}_{-9.5}$	$86\,^{+21}_{-11}|| - 2\,^{+16}_{-17}$
β [°]	$22.56\,^{+0.47}_{-0.40}$	$22.56\,^{+1.12}_{-0.81}$	$22.6\,^{+1.8}_{-1.2}$
β [°] (meas. not in the fit)	$23.7\,^{+1.3}_{-1.2}$	$23.7\,^{+2.7}_{-1.8}$	$23.7\,^{+3.4}_{-2.4}$
β [°] (dir. meas.)	$22.14\,^{+0.69}_{-0.67}$	$22.1\,^{+1.4}_{-1.3}$	$22.1\,^{+2.1}_{-2.0}$
γ [°]	$65.80\,^{+0.94}_{-1.29}$	$65.8\,^{+1.9}_{-4.0}$	$65.8\,^{+2.9}_{-5.2}$
γ [°] (meas. not in the fit)	$65.66\,^{+0.90}_{-2.65}$	$65.7\,^{+1.9}_{-4.5}$	$65.7\,^{+2.9}_{-5.5}$
γ [°] (dir. meas.)	$72.1\,^{+5.4}_{-5.7}$	$72\,^{+10}_{-12}$	$72\,^{+15}_{-21}$
R_u	$0.3837\,^{+0.0078}_{-0.0064}$	$0.384\,^{+0.018}_{-0.013}$	$0.384\,^{+0.028}_{-0.020}$
R_t	$0.9124\,^{+0.0064}_{-0.0100}$	$0.912\,^{+0.013}_{-0.027}$	$0.912\,^{+0.019}_{-0.036}$
$\bar{\rho}_s$	$-0.00840\,^{+0.00038}_{-0.00047}$	$-0.00840\,^{+0.00072}_{-0.00139}$	$-0.0084\,^{+0.0011}_{-0.0019}$
$\bar{\eta}_s$	$-0.01866\,^{+0.00036}_{-0.00041}$	$-0.01866\,^{+0.00088}_{-0.00092}$	$-0.0187\,^{+0.0013}_{-0.0014}$
$\beta_s \equiv -\arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right)$ [rad]	$0.01848\,^{+0.00042}_{-0.00036}$	$0.01848\,^{+0.00093}_{-0.00087}$	$0.0185\,^{+0.0014}_{-0.0013}$
$\sin 2\beta_s$	$0.03698\,^{+0.00081}_{-0.00070}$	$0.0370\,^{+0.0018}_{-0.0017}$	$0.0370\,^{+0.0028}_{-0.0025}$

(!) means that the quantity was not included in the fit, || indicates the union of the confidence intervals considered.

Observable	central ± CL ≡ 1σ	± CL ≡ 2σ	± CL ≡ 3σ
$\|V_{ud}\|$	$0.974390\,^{+0.000014}_{-0.000058}$	$0.974390\,^{+0.000027}_{-0.000140}$	$0.974390\,^{+0.000041}_{-0.000225}$
$\|V_{us}\|$	$0.224834\,^{+0.000252}_{-0.000059}$	$0.22483\,^{+0.00060}_{-0.00012}$	$0.22483\,^{+0.00097}_{-0.00018}$
$\|V_{ub}\|$	$0.003683\,^{+0.000075}_{-0.000061}$	$0.00368\,^{+0.00018}_{-0.00013}$	$0.00368\,^{+0.00028}_{-0.00018}$
$\|V_{cd}\|$	$0.224701\,^{+0.000254}_{-0.000058}$	$0.22470\,^{+0.00061}_{-0.00012}$	$0.22470\,^{+0.00097}_{-0.00018}$
$\|V_{cs}\|$	$0.973539\,^{+0.000038}_{-0.000060}$	$0.973539\,^{+0.000061}_{-0.000142}$	$0.973539\,^{+0.000081}_{-0.000229}$
$\|V_{cb}\|$	$0.04162\,^{+0.00026}_{-0.00080}$	$0.04162\,^{+0.00055}_{-0.00117}$	$0.04162\,^{+0.00078}_{-0.00142}$
$\|V_{td}\|$	$0.008545\,^{+0.000075}_{-0.000157}$	$0.00855\,^{+0.00016}_{-0.00029}$	$0.00855\,^{+0.00024}_{-0.00038}$
$\|V_{ts}\|$	$0.04090\,^{+0.00026}_{-0.00076}$	$0.04090\,^{+0.00053}_{-0.00114}$	$0.04090\,^{+0.00075}_{-0.00138}$
$\|V_{tb}\|$	$0.999127\,^{+0.000032}_{-0.000012}$	$0.999127\,^{+0.000047}_{-0.000023}$	$0.999127\,^{+0.000058}_{-0.000033}$
$\|V_{ud}\|$ (meas. not in the fit)	$0.974395\,^{+0.000062}_{-0.000061}$	$0.97439\,^{+0.00011}_{-0.00014}$	$0.97439\,^{+0.00016}_{-0.00023}$
$\|V_{us}\|$ (meas. not in the fit)	$0.224833\,^{+0.001336}_{-0.000061}$	$0.22483\,^{+0.00191}_{-0.00061}$	$0.22483\,^{+0.00198}_{-0.00018}$
$\|V_{ub}\|$ (meas. not in the fit)	$0.003624\,^{+0.000109}_{-0.000084}$	$0.00362\,^{+0.00023}_{-0.00017}$	$0.00362\,^{+0.00034}_{-0.00024}$
$\|V_{cb}\|$ (meas. not in the fit)	$0.04191\,^{+0.00037}_{-0.00095}$	$0.04191\,^{+0.00067}_{-0.00144}$	$0.04191\,^{+0.00097}_{-0.00180}$
Δm_d [ps^{-1}] (meas. not in the fit)	$0.577\,^{+0.036}_{-0.039}$	$0.577\,^{+0.065}_{-0.077}$	$0.577\,^{+0.095}_{-0.116}$
Δm_s [ps^{-1}] (meas. not in the fit)	$16.70\,^{+0.73}_{-0.45}$	$16.70\,^{+1.89}_{-0.77}$	$16.7\,^{+2.9}_{-1.1}$
$\|\epsilon_K\|$ [10^{-3}] (meas. not in the fit)	$2.10\,^{+0.27}_{-0.20}$	$2.10\,^{+0.38}_{-0.38}$	$2.10\,^{+0.47}_{-0.46}$
m_t [GeV/c^2] (meas. not in the fit)	$158.6\,^{+6.4}_{-2.5}$	$158.6\,^{+14.9}_{-4.6}$	$158.6\,^{+18.3}_{-6.6}$
B_K (lattice value not in the fit)	$0.782\,^{+0.105}_{-0.063}$	$0.782\,^{+0.181}_{-0.093}$	$0.78\,^{+0.23}_{-0.12}$
f_{B_s}/f_{B_d} (lattice value not in the fit)	$1.263\,^{+0.045}_{-0.030}$	$1.263\,^{+0.079}_{-0.077}$	$1.26\,^{+0.11}_{-0.11}$
f_{B_s} (lattice value not in the fit)	$0.2249\,^{+0.0081}_{-0.0041}$	$0.2249\,^{+0.0113}_{-0.0063}$	$0.2249\,^{+0.0137}_{-0.0083}$
B_{B_s}/B_{B_d} (lattice value not in the fit)	$1.146\,^{+0.056}_{-0.066}$	$1.15\,^{+0.11}_{-0.14}$	$1.15\,^{+0.16}_{-0.21}$
B_{B_s} (lattice value not in the fit)	$1.307\,^{+0.062}_{-0.057}$	$1.307\,^{+0.090}_{-0.080}$	$1.307\,^{+0.112}_{-0.099}$

(!) means that the quantity was not included in the fit, || indicates the union of the confidence intervals considered.

Observable	central $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
$\mathcal{B}(B^+ \to \tau\nu)$ $[10^{-4}]$	$0.851^{+0.039}_{-0.038}$	$0.851^{+0.095}_{-0.069}$	$0.851^{+0.148}_{-0.095}$
$\mathcal{B}(B^+ \to \tau\nu)$ $[10^{-4}]$ (meas. not in the fit)	$0.838^{+0.039}_{-0.029}$	$0.838^{+0.092}_{-0.059}$	$0.838^{+0.149}_{-0.086}$
$\mathcal{B}(B^+ \to \mu\nu)$ $[10^{-6}]$	$0.378^{+0.019}_{-0.013}$	$0.378^{+0.045}_{-0.026}$	$0.378^{+0.070}_{-0.038}$
$\mathcal{B}(B^+ \to e\nu)$ $[10^{-11}]$	$0.885^{+0.048}_{-0.032}$	$0.885^{+0.110}_{-0.061}$	$0.885^{+0.165}_{-0.090}$
$\mathcal{B}(B_d \to e^+e^-)$ $[10^{-15}]$	$2.200^{+0.116}_{-0.069}$	$2.20^{+0.19}_{-0.14}$	$2.20^{+0.24}_{-0.18}$
$\mathcal{B}(B_d \to \mu^+\mu^-)$ $[10^{-11}]$	$9.39^{+0.62}_{-0.32}$	$9.39^{+0.83}_{-0.57}$	$9.39^{+1.03}_{-0.76}$
$\mathcal{B}(B_s \to e^+e^-)$ $[10^{-14}]$	$7.49^{+0.36}_{-0.28}$	$7.49^{+0.57}_{-0.37}$	$7.49^{+0.67}_{-0.45}$
$\mathcal{B}(B_s \to \mu^+\mu^-)$ $[10^{-9}]$	$3.20^{+0.18}_{-0.12}$	$3.20^{+0.24}_{-0.16}$	$3.20^{+0.29}_{-0.19}$
$\mathcal{B}(B_s \to \mu^+\mu^-)$ $[10^{-9}]$ (meas. not in the fit)	$3.27^{+0.12}_{-0.16}$	$3.27^{+0.18}_{-0.23}$	$3.27^{+0.22}_{-0.26}$
$\mathcal{B}(D_s \to \tau^+\nu)$	$0.05224^{+0.00011}_{-0.00057}$	$0.05224^{+0.00022}_{-0.00074}$	$0.05224^{+0.00034}_{-0.00086}$
$\mathcal{B}(D_s \to \tau^+\nu)$ (meas. not in the fit)	$0.05223^{+0.00012}_{-0.00063}$	$0.05223^{+0.00023}_{-0.00075}$	$0.05223^{+0.00034}_{-0.00087}$
$\mathcal{B}(D_s \to \mu^+\nu)$ $[10^{-2}]$	$0.5360^{+0.0011}_{-0.0059}$	$0.5360^{+0.0023}_{-0.0075}$	$0.5360^{+0.0035}_{-0.0088}$
$\mathcal{B}(D_s \to \mu^+\nu)$ $[10^{-2}]$ (meas. not in the fit)	$0.5357^{+0.0014}_{-0.0064}$	$0.5357^{+0.0025}_{-0.0076}$	$0.5357^{+0.0037}_{-0.0088}$
$\mathcal{B}(D \to \tau^+\nu)$ $[10^{-3}]$	$1.069^{+0.015}_{-0.017}$	$1.069^{+0.036}_{-0.036}$	$1.069^{+0.051}_{-0.049}$
$\mathcal{B}(D \to \tau^+\nu)$ $[10^{-3}]$ (meas. not in the fit)	$1.069^{+0.014}_{-0.017}$	$1.069^{+0.031}_{-0.036}$	$1.069^{+0.051}_{-0.049}$
$\mathcal{B}(D \to \mu^+\nu)$ $[10^{-3}]$	$0.4007^{+0.0026}_{-0.0045}$	$0.4007^{+0.0065}_{-0.0059}$	$0.4007^{+0.0087}_{-0.0069}$
$\mathcal{B}(D \to \mu^+\nu)$ $[10^{-3}]$ (meas. not in the fit)	$0.4050^{+0.0016}_{-0.0073}$	$0.4050^{+0.0031}_{-0.0099}$	$0.4050^{+0.0049}_{-0.0111}$
$\mathcal{B}(K \to \mu^+\nu)$	$0.63550^{+0.00091}_{-0.00088}$	$0.6355^{+0.0019}_{-0.0018}$	$0.6355^{+0.0029}_{-0.0028}$
$\mathcal{B}(K \to \mu^+\nu)$ (meas. not in the fit)	$0.6353^{+0.0017}_{-0.0014}$	$0.6353^{+0.0037}_{-0.0036}$	$0.6353^{+0.0057}_{-0.0060}$
$\mathcal{B}(K \to e^+\nu)$ $[10^{-4}]$	$0.15699^{+0.00023}_{-0.00021}$	$0.15699^{+0.00047}_{-0.00043}$	$0.15699^{+0.00071}_{-0.00067}$
$\mathcal{B}(K \to e^+\nu)$ $[10^{-4}]$ (meas. not in the fit)	$0.15685^{+0.00023}_{-0.00022}$	$0.15685^{+0.00048}_{-0.00047}$	$0.15685^{+0.00073}_{-0.00074}$
$\mathcal{B}(\tau^+ \to K\nu)$ $[10^{-2}]$	$0.71666^{+0.00106}_{-0.00095}$	$0.7167^{+0.0022}_{-0.0020}$	$0.7167^{+0.0032}_{-0.0030}$
$\mathcal{B}(\tau^+ \to K\nu)$ $[10^{-2}]$ (meas. not in the fit)	$0.7169^{+0.0011}_{-0.0010}$	$0.7169^{+0.0022}_{-0.0020}$	$0.7169^{+0.0033}_{-0.0031}$

(!) means that the quantity was not included in the fit, || indicates the union of the confidence intervals considered.

References

[1] I.S. Towner and J.C. Hardy, J. C. Hardy and I. S. Towner, arXiv:1807.01146 [nucl-ex], presented at CIPANP 2018, and updated at the Amherst Workshop *Current and Future Status of the First-Row CKM Unitarity* in May 2019 https://www.physics.umass.edu/acfi/sites/acfi/files/slides/hardy_amherst_workshop_talk_0.pdf

[2] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98** (2018) no.3, 030001.

[3] Y. S. Amhis *et al.* [HFLAV Collaboration], arXiv:1909.12524 [hep-ex]. and their updates on https://hflav.web.cern.ch and references therein.

[4] See sec. 2.

[5] We use the same inputs as in Ref. [3], but we separate the statistical and systematic uncertainties, leading to the inclusive $|V_{ub}| = (4.25 \pm 0.14 \pm 0.33) \times 10^{-3}$ and exclusive $|V_{ub}| = (3.70 \pm 0.10 \pm 0.21) \times 10^{-3}$ Our average of the two values is obtained using the Edudacted Rfit approach.

[6] We use the same inputs as in Ref. [3], but we separate the statistical and systematic uncertainties, leading to the inclusive $|V_{cb}| = (42.2 \pm 0.4 \pm 0.5) \times 10^{-3}$ and exclusive $|V_{cb}| = (39.8 \pm 0.6 \pm 0.5) \times 10^{-3}$ Our average of the two values is obtained using the Edudacted Rfit approach.

[7] R. Aaij *et al.* [LHCb Collaboration], Nature Phys. **11** (2015) [arXiv:1504.01568 [hep-ex]].

[8] R. Aaij *et al.* [LHCb Collaboration], Nature Phys. **11**, 743 (2015) [arXiv:1504.01568 [hep-ex]].
W. Detmold, C. Lehner and S. Meinel, Phys. Rev. D **92**, no. 3, 034503 (2015) [arXiv:1503.01421 [hep-lat]].

[9] J. P. Lees *et al.*, Phys.Rev. **D 88** 031102 (2013) [arXiv:1207.0698 [hep-ex]],
B. Aubert *et al.*, Phys. Rev. **D 81**, 051101(R) (2010),
I. Adachi *et al.*, Phys. Rev. Lett. 110, 131801 (2013), arXiv:1208.4678 [hep-ex],
B. Kronenbitter *et al.*, arXiv:1503.05613 [hep-ex].

[10] J. Chen, *Leptonic and semi-leptonic decays of charmed mesons at BESIII*, talk at the ICHEP 2018 conference (Seoul, South Korea). https://indico.cern.ch/event/686555/contributions/2982776/

[11] V. Khachatryan *et al.* [CMS and LHCb Collaborations], Nature **522** (2015) 68

[12] BaBar collaboration, Phys. Rev. D75, 012008 (2007).

[13] BaBar collaboration, Phys.Rev. D87 (2013) 052009.

[14] BaBar collaboration, Phys. Rev. D76, 091102 (2007).

[15] Belle collaboration, Phys. Rev. D87, 031103 (2013).

[16] I. Adachi *et al.* [Belle Collaboration], Phys. Rev. D **88** (2013) no.9, 092003

[17] Belle Collaboration, Phys. Rev. D **96** (2017) no.3, 032007.

[18] CDF collaboration, Phys. Rev. Lett. 106, 181802 (2011).

[19] Cleo collaboration, Phys. Rev. D68, 052002 (2003).

[20] LHCb Collaboration], Phys. Rev. D **98** (2018) no.3, 032004

[21] BaBar collaboration, Phys. Rev. D76, 052007 (2007).

[22] BaBar collaboration, Phys. Rev. Lett. 102, 141802 (2009).

[23] BaBar collaboration, Phys. Rev. D78, 071104 (2008).

[24] Belle collaboration,Phys. Rev. D93, 032010 (2016).

[25] Belle collaboration, Phys. Rev. Lett. 91, 221801 (2003).

[26] Belle collaboration, Phys. Rev. D89 072008 (2014).

[27] LHCb collaboration, arXiv:hep-ex/1503.07770 (2015).

[28] Babar collaboration, Phys. Rev. D 88, 012003 (2013).

[29] Belle collaboration, Phys. Rev. Lett. 98, 221602 (2007).

[30] P. del Amo Sanchez *et al.*[BaBar Collaboration], Phys. Rev. D **82** (2010) 072004.
   B. Aubert *et al* [BaBar Collaboration] Phys. Rev. D **78** (2008) 092002.
   B. Aubert *et al* [BaBar Collaboration], Phys. Rev. D **80** (2009) 092001.
   K. Trabelsi for the Belle Collaboration, arXiv:1301.2033, preliminary results.
   R. Aaij *et al* [LHCb Collaboration] Physics Letters B 760 (2016).
   R. Aaij *et al* [LHCb Collaboration], Phys. Rev. D 91, 112014 (2015).

[31] P. del Amo Sanchez *et al* [BaBar Collaboration], Phys.Rev. **D82** (2010) 072006.
   Y. Horii *et al* [Belle Collaboration], Phys.Rev.Lett. **106** (2011) 231803.
   K. Trabelsi for the Belle Collaboration, arXiv:1301.2033, preliminary results.
   R. Aaij *et al* [LHCb Collaboration] Physics Letters B 760 (2016).
   R. Aaij *et al* [LHCb Collaboration], Phys. Rev. D 91, 112014 (2015).
   M. Nayak *et al.* [Belle Collaboration], Phys. Rev. D **88** (2013) 9, 091104

[32] R. Aaij *et al* [LHCb Collaboration],Phys. Rev. D90 (2014) 112002

[33] P. del Amo Sanchez *et al* [BaBar Collaboration], Phys.Rev.Lett. 105 (2010) 121801.
   A. Poluektov *et al* [Belle Collaboration], Phys.Rev. D81 (2010) 112002.
   R. Aaij *et al* [LHCb Collaboration], JHEP 10 (2014) 097.
   R. Aaij *et al* [LHCb Collaboration], Phys. Rev. D 92, 112005 (2015).

[34] R. Aaij *et al* [LHCb Collaboration], JHEP08(2016)137

[35] The world average of the top quark mass measurements performed at the LHC [36] interpreted as a pole mass is translated into $m_t(m_t)$ in the $\overline{\text{MS}}$ at 1-loop order.

[36] M. Aaboud *et al.* [ATLAS Collaboration], Eur. Phys. J. C **79** (2019) no.4, 290 [arXiv:1810.01772 [hep-ex]].

V. Khachatryan *et al.* [CMS Collaboration], Phys. Rev. D **93** (2016) no.7, 072004 [arXiv:1509.04044 [hep-ex]].

[37] A. J. Buras and D. Guadagnoli, Phys. Rev. D **78** (2008) 033005 [arXiv:0805.3887 [hep-ph]].

A. J. Buras, D. Guadagnoli and G. Isidori, Phys. Lett. B **688** (2010) 309 [arXiv:1002.3612 [hep-ph]].

[38] A. Lenz, U. Nierste, J. Charles, S. Descotes-Genon, A. Jantsch, C. Kaufhold, H. Lacker and S. Monteil *et al.*, Phys. Rev. D **83** (2011) 036004 [arXiv:1008.1593 [hep-ph]].

[39] J. Brod, M. Gorbahn and E. Stamou, arXiv:1911.06822 [hep-ph].

[40] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[41] A. Lenz, private communication (2010).

[42] S. Aoki *et al.* [Flavour Lattice Averaging Group], arXiv:1902.08191 [hep-lat]. and web update on: `http://flag.unibe.ch`

[43] B. Blossier *et al.*, JHEP **0907** (2009) 043 [arXiv:0904.0954 [hep-lat]].

[44] E. Follana, C. T. H. Davies, G. P. Lepage and J. Shigemitsu [HPQCD Collaboration and UKQCD Collaboration], Phys. Rev. Lett. **100**, 062002 (2008) [arXiv:0706.1726 [hep-lat]].

[45] A. Bazavov *et al.* [MILC Collaboration], PoS LATTICE **2010** (2010) 074 [arXiv:1012.0868 [hep-lat]].

[46] R. J. Dowdall, C. T. H. Davies, G. P. Lepage and C. McNeile, Phys. Rev. D **88** (2013) 074504 [arXiv:1303.1670 [hep-lat]].

[47] N. Carrasco, P. Dimopoulos, R. Frezzotti, P. Lami, V. Lubicz, F. Nazzaro, E. Picca and L. Riggio *et al.*, Phys. Rev. D **91** (2015) 5, 054507 [arXiv:1411.7908 [hep-lat]].

[48] S. Dürr *et al.*, Phys. Rev. D **81** (2010) 054507 [arXiv:1001.4692 [hep-lat]].

[49] S. Dürr *et al.*, Phys. Rev. D **95** (2017) no.5, 054513 [arXiv:1601.05998 [hep-lat]].

[50] V. G. Bornyakov *et al.* [QCDSFUKQCD Collaboration], Phys. Lett. B **767** (2017) 366 [arXiv:1612.04798 [hep-lat]].

[51] A. Bazavov *et al.*, Phys. Rev. D **98** (2018) no.7, 074512 [arXiv:1712.09262 [hep-lat]].

[52] N. Carrasco *et al.* [ETM Collaboration], JHEP **1403** (2014) 016 [arXiv:1308.1851 [hep-lat]].

[53] C. T. H. Davies, C. McNeile, E. Follana, G. P. Lepage, H. Na and J. Shigemitsu, Phys. Rev. D **82** (2010) 114504 [arXiv:1008.4018 [hep-lat]].

[54] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaboration], Phys. Rev. D **85** (2012) 114506 [arXiv:1112.3051 [hep-lat]].

[55] Y. B. Yang *et al.*, Phys. Rev. D **92** (2015) no.3, 034517 [arXiv:1410.3343 [hep-lat]].

[56] P. A. Boyle, L. Del Debbio, A. Jttner, A. Khamseh, F. Sanfilippo and J. T. Tsang, JHEP **1712** (2017) 008 [arXiv:1701.02644 [hep-lat]].

[57] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **86** (2012) 054510 [arXiv:1206.4936 [hep-lat]].

[58] F. Bernardoni, B. Blossier, J. Bulava, M. Della Morte, P. Fritzsch, N. Garron, A. Gérardin and J. Heitger *et al.*, Phys. Lett. B **735** (2014) 349 [arXiv:1404.3590 [hep-lat]].

[59] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel and G. P. Lepage, Phys. Rev. D **85** (2012) 031503 [arXiv:1110.4510 [hep-lat]].

[60] N. H. Christ, J. M. Flynn, T. Izubuchi, T. Kawanai, C. Lehner, A. Soni, R. S. Van de Water and O. Witzel, Phys. Rev. D **91** (2015) 5, 054502 [arXiv:1404.4670 [hep-lat]].

[61] R. J. Dowdall *et al.* [HPQCD Collaboration], arXiv:1302.2644 [hep-lat].

[62] A. Bussone *et al.* [ETM Collaboration], Phys. Rev. D **93** (2016) no.11, 114505 [arXiv:1603.04306 [hep-lat]].

[63] C. Hughes, C. T. H. Davies and C. J. Monahan, Phys. Rev. D **97** (2018) no.5, 054509 [arXiv:1711.09981 [hep-lat]].

[64] H. Na, C. J. Monahan, C. T. H. Davies, R. Horgan, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **86** (2012) 034506 [arXiv:1202.4914 [hep-lat]].

[65] J. L. Rosner, S. Stone and R. S. Van de Water, arXiv:1509.02220 [hep-ph].

[66] Y. Aoki, T. Ishikawa, T. Izubuchi, C. Lehner and A. Soni, Phys. Rev. D **91** (2015) no.11, 114505 [arXiv:1406.6192 [hep-lat]].

[67] V. Lubicz, F. Mescia, S. Simula, C. Tarantino and f. t. E. Collaboration, Phys. Rev. D **80** (2009) 111502 [arXiv:0906.4728 [hep-lat]].

[68] A. Bazavov *et al.*, Phys. Rev. D **87** (2013) 073012 [arXiv:1212.4993 [hep-lat]].

[69] P. A. Boyle *et al.* [RBC/UKQCD Collaboration], JHEP **1506** (2015) 164 [arXiv:1504.01692 [hep-lat]].

[70] A. Bazavov *et al.*, Phys. Rev. Lett. **112** (2014) no.11, 112001 [arXiv:1312.1228 [hep-ph]].

[71] N. Carrasco, P. Lami, V. Lubicz, L. Riggio, S. Simula and C. Tarantino, Phys. Rev. D **93** (2016) no.11, 114512 [arXiv:1602.04113 [hep-lat]].

[72] H. Na, C. T. H. Davies, E. Follana, J. Koponen, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **84** (2011) 114505 [arXiv:1109.1501 [hep-lat]].

[73] V. Lubicz *et al.* [ETM Collaboration], Phys. Rev. D **96** (2017) no.5, 054514 [arXiv:1706.03017 [hep-lat]].

[74] H. Na, C. T. H. Davies, E. Follana, G. P. Lepage and J. Shigemitsu, Phys. Rev. D **82**, 114506 (2010) [arXiv:1008.4562 [hep-lat]].

[75] W. Detmold, C. Lehner and S. Meinel, Phys. Rev. D **92** (2015) no.3, 034503 [arXiv:1503.01421 [hep-lat]].

[76] V. Bertone *et al.* [ETM Collaboration], JHEP **1303** (2013) 089 Erratum: [JHEP **1307** (2013) 143] [arXiv:1207.1287 [hep-lat]].

[77] J. Laiho and R. S. Van de Water, PoS LATTICE **2011** (2011) 293 [arXiv:1112.4861 [hep-lat]].

[78] S. Durr, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch and T. Lippert *et al.*, Phys. Lett. B **705** (2011) 477 [arXiv:1106.3230 [hep-lat]].

[79] T. Blum *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **93** (2016) no.7, 074505 [arXiv:1411.7017 [hep-lat]].

[80] B. J. Choi *et al.* [SWME Collaboration], Phys. Rev. D **93** (2016) no.1, 014511 [arXiv:1509.00592 [hep-lat]].

[81] N. Carrasco *et al.* [ETM Collaboration], Phys. Rev. D **92** (2015) no.3, 034516 [arXiv:1505.06639 [hep-lat]].

[82] E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu and M. Wingate [HPQCD Collaboration], Phys. Rev. D **80**, 014503 (2009) [arXiv:0902.1815 [hep-lat]].

[83] A. Bazavov *et al.* [Fermilab Lattice and MILC Collaborations], Phys. Rev. D **93** (2016) no.11, 113016 [arXiv:1602.03560 [hep-lat]].

[84] C. McNeile, C. T. H. Davies, E. Follana, K. Hornbostel and G. P. Lepage, Phys. Rev. D **82** (2010) 034512 [arXiv:1004.4285 [hep-lat]].

[85] C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis and G. Koutsou, Phys. Rev. D **90** (2014) no.7, 074501 [arXiv:1406.4310 [hep-lat]].

[86] B. Chakraborty *et al.*, Phys. Rev. D **91** (2015) no.5, 054508 [arXiv:1408.4169 [hep-lat]].

[87] A. T. Lytle *et al.* [HPQCD Collaboration], Phys. Rev. D **98** (2018) no.1, 014513 [arXiv:1805.06225 [hep-lat]].

[88] A. Bazavov *et al.* [Fermilab Lattice and MILC and TUMQCD Collaborations], Phys. Rev. D **98** (2018) no.5, 054517 [arXiv:1802.04248 [hep-lat]].