UPDATED RESULTS ON THE CKM MATRIX

Including results presented up to
Summer 23

P r e l i m i n a r y

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The CKMfitter Group

Abstract

This document provides the collection of up-to-date inputs to the global CKM analysis, and numerical results obtained with the use of the fit package CKMfitter. The statistical method employed is the frequentist approach, using the *R*fit model to treat theoretical uncertainties. Detailed background information on the methodology and the treatment of experimental and theoretical uncertainties is provided in:

CP VIOLATION AND THE CKM MATRIX:
ASSESSING THE IMPACT OF THE ASYMMETRIC *B* FACTORIES
By CKMfitter Group
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The CKMfitter Group

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1 Inputs

2 Lattice QCD averages

Several hadronic inputs are required for the fits presented by CKMfitter, and we mostly rely on lattice QCD simulations to estimate these quantities. The presence of results from different collaborations with various statistics and systematics make it all the more necessary to combine them in a careful way. We explain below the procedure that we have chosen to determine these lattice averages.

3 Method of averaging

We collect the relevant calculations of the quantity that we are interested in: we take results with 2, 2+1 or 2+1+1 dynamical fermions, even those from proceedings without a companion article. For data prior to the publication of the latest FLAG review [65], we followed their prescriptions to discard results of insufficient quality when lattice QCD data satisfying their quality criteria are available.

In these results, we separate the error estimates into a Gaussian part and a flat part that is treated à la Rfit. The Gaussian part collects the uncertainties from purely statistical origin, but also the systematics that can be controlled and treated in a similar way (e.g., interpolation or fitting in some cases). The remaining systematics constitute the Rfit error. If there are several sources of error in the Rfit category, we add them linearly [1].

The Rfit model is simple but also very strict. It amounts to assuming that the theoretical uncertainty is rigorously constrained by a mathematical bound that is our only piece of information. If Rfit is taken stricto sensu and the individual likelihoods are combined in the usual way (by multiplication), the final uncertainty can be underestimated, in particular in the case of marginally compatible values.

We correct this effect by adopting the following averaging recipe. The central value is obtained by combining the whole likelihoods. Then we combine the Gaussian uncertainties by combining likelihoods restricted to their Gaussian part. Finally we assign to this combination the smallest of the individual Rfit uncertainties. The underlying idea is twofold:

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
- this best estimate should not be penalized by less precise methods (as it would happen be the case if one would take the dispersion of the individual central values as a guess of the combined theoretical uncertainty).

It should be stressed that the concept of a theoretical uncertainty is ill-defined, and the combination of them even more. Thus our approach is only one among the alternatives that can be found in the literature. In contrast to some of the latter, ours is algorithmic and can be reproduced.

[1] keeping in mind that in many papers in the literature, this combination is done in quadrature and the splitting between different sources is not published.

Parameter	Value ± Error(s)	Reference	Errors GS	Errors TH	Tree-level fit
$\lvert V_{ud}\rvert$ (nuclei)	$0.97373 \pm 0.00009 \pm 0.00053$	[1]	⋆	⋆	⋆
$\lvert V_{us}\rvert f_+^{K\to\pi}(0)$	0.21635 ± 0.00038	[62]	⋆	-	⋆
$\lvert V_{cd}\rvert$ (νN)	0.230 ± 0.011	[2]	⋆	-	⋆
$\lvert V_{cs}\rvert$ ($W \to c\bar{s}$)	0.967 ± 0.011	[60]	⋆	-	⋆
$\lvert V_{ub}\rvert$ (semileptonic)	$(3.86 \pm 0.07 \pm 0.12) \times 10^{-3}$	[6]	⋆	⋆	⋆
$\lvert V_{cb}\rvert$ (semileptonic)	$(41.22 \pm 0.24 \pm 0.37) \times 10^{-3}$	[7]	⋆	⋆	⋆
$\lvert V_{ub}\rvert/\lvert V_{cb}\rvert$ (inclusive semileptonic)	$0.100 \pm 0.006 \pm 0.003$	[63]	⋆	⋆	⋆
$\mathcal{B}(\Lambda_b^0 \to p\mu^-\overline{\nu}_\mu)_{q^2>15}/\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\overline{\nu}_\mu)_{q^2>7}$	$(0.947 \pm 0.081) \times 10^{-2}$	[14, 15]	⋆	-	⋆
$\mathcal{B}(B_s^0 \to K^-\mu^+\nu_\mu)_{q^2>7}/\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)$	$(3.25 \pm 0.28) \times 10^{-3}$	[16]	⋆	-	⋆
$\mathcal{B}(B^- \to \tau^-\overline{\nu}_\tau)$	$(1.09 \pm 0.24) \times 10^{-4}$	[3, 17]	⋆	-	⋆
$\mathcal{B}(D_s^- \to \mu^-\overline{\nu}_\mu)$	$(5.43 \pm 0.16) \times 10^{-3}$	[3]	⋆	-	⋆
$\mathcal{B}(D_s^- \to \tau^-\overline{\nu}_\tau)$	$(5.32 \pm 0.10) \times 10^{-2}$	[61]	⋆	-	⋆
$\mathcal{B}(D^- \to \mu^-\overline{\nu}_\mu)$	$(3.77 \pm 0.17) \times 10^{-4}$	[3]	⋆	-	⋆
$\mathcal{B}(D^- \to \tau^-\overline{\nu}_\tau)$	$(1.20 \pm 0.27) \times 10^{-3}$	[3]	⋆	-	⋆
$\mathcal{B}(K^- \to e^-\overline{\nu}_e)$	$(1.582 \pm 0.007) \times 10^{-5}$	[2]	⋆	-	⋆
$\mathcal{B}(K^- \to \mu^-\overline{\nu}_\mu)$	0.6356 ± 0.0011	[2]	⋆	-	⋆
$\mathcal{B}(\tau^- \to K^-\overline{\nu}_\tau)$	$(0.6986 \pm 0.0085) \times 10^{-2}$	[3]	⋆	-	⋆
$\mathcal{B}(K^- \to \mu^-\overline{\nu}_\mu)/\mathcal{B}(\pi^- \to \mu^-\overline{\nu}_\mu)$	1.3367 ± 0.0028	[2]	⋆	-	⋆
$\mathcal{B}(\tau^- \to K^-\overline{\nu}_\tau)/\mathcal{B}(\tau^- \to \pi^-\overline{\nu}_\tau)$	$(6.437 \pm 0.092) \times 10^{-2}$	[3]	⋆	-	⋆
$\mathcal{B}(B_s \to \mu\mu)$	$(3.23 \pm 0.27) \times 10^{-9}$	[3]	⋆	-	-
$\lvert V_{cd}\rvert f_+^{D\to\pi}(0)$	0.1426 ± 0.0018	[3]	⋆	-	⋆
$\lvert V_{cs}\rvert f_+^{D\to K}(0)$	0.7180 ± 0.0033	[3]	⋆	-	⋆
$\lvert\varepsilon_K\rvert$	$(2.228 \pm 0.011) \times 10^{-3}$	[2]	⋆	-	-
Δm_d	(0.5065 ± 0.0019) ps^{-1}	[3]	⋆	-	-
Δm_s	(17.765 ± 0.006) ps^{-1}	[20]	⋆	-	-
$\sin(2\beta)_{[c\bar{c}]}$	0.708 ± 0.011	[3]	⋆	-	-
$(\phi_s)_{[b\to c\bar{c}s]}$	-0.039 ± 0.016	[3]	⋆	-	-
$S_{\pi\pi}^{+-}$, $C_{\pi\pi}^{+-}$, $C_{\pi\pi}^{00}$,$\mathcal{B}_{\pi\pi}$ all charges	Inputs to isospin analysis	[21–32]	⋆	-	-
$S_{\rho\rho,L}^{+-}$, $C_{\rho\rho,L}^{+-}$, $S_{\rho\rho}^{00}$, $C_{\rho\rho}^{00}$,$\mathcal{B}_{\rho\rho,L}$ all charges	Inputs to isospin analysis	[33–41]	⋆	-	-
$B^0 \to (\rho\pi)^0 \to 3\pi$	Time-dependent Dalitz analysis	[42, 43]	⋆	-	-
γ ($B \to D^{(*)}K^{(*)}$; GGSZ, GLW, ADS methods)	$(65.9^{+3.3}_{-3.5})^{\circ}$	[3]	⋆	-	⋆

Table 1: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. Two columns indicate* R*fit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the "GS" column by a star; measurements or parameters that have systematic theoretical errors are marked in the "TH" column by a star. The last column indicates by a star which observables are included in the tree-level fit observable. Upper part: experimental determinations of the CKM matrix elements. Lower part: CP-violation and mixing observables.*

Parameter	Value ± Error(s)	Reference	Errors GS	Errors TH
$\overline{m}_c(\overline{m}_c)$	$(1.2977 \pm 0.0013 \pm 0.0120)$ GeV	[5]	⋆	⋆
$\overline{m}_t(\overline{m}_t)$	$(165.26 \pm 0.11 \pm 0.30)$ GeV	[49]	⋆	⋆
$\alpha_s(m_Z)$	$0.1179 \pm 0 \pm 0.0009$	[2]	-	⋆
$\hat{B}_K$	$0.7567 \pm 0.0020 \pm 0.0123$	[5]	⋆	⋆
κ_ϵ	$0.940 \pm 0.013 \pm 0.023$	[51, 52]	⋆	⋆
η_{ut}	$0.404 \pm 0 \pm 0.007$	[53, 54]	-	⋆
η_{tt}	$0.54 \pm 0 \pm 0.03$	[53, 55]	-	⋆
$\eta_B(\overline{\text{MS}})$	$0.5510 \pm 0 \pm 0.0022$	[56, 57]	-	⋆
f_{B_s}	$(228.75 \pm 0.69 \pm 1.87)$ MeV	[5]	⋆	⋆
$\hat{B}_s$	$1.313 \pm 0.012 \pm 0.030$	[5]	⋆	⋆
f_{B_s}/f_{B_d}	$1.2118 \pm 0.0020 \pm 0.0058$	[5]	⋆	⋆
$\hat{B}_s/\hat{B}_d$	$1.007 \pm 0.010 \pm 0.014$	[5]	⋆	⋆
f_K	$(155.57 \pm 0.17 \pm 0.57)$ MeV	[5]	⋆	⋆
f_K/f_π	$1.1973 \pm 0.0007 \pm 0.0014$	[5]	⋆	⋆
f_{D_s}	$(249.23 \pm 0.27 \pm 0.65)$ MeV	[5]	⋆	⋆
f_{D_s}/f_D	$1.1782 \pm 0.0006 \pm 0.0033$	[5]	⋆	⋆
$f_+^{K\to\pi}(0)$	$0.9675 \pm 0.0011 \pm 0.0023$	[5]	⋆	⋆
$f_+^{D\to\pi}(0)$	$0.624 \pm 0.004 \pm 0.006$	[5]	⋆	⋆
$f_+^{D\to K}(0)$	$0.742 \pm 0.002 \pm 0.004$	[5]	⋆	⋆
$\zeta(\Lambda_b^0 \to p\mu^-\overline{\nu}_\mu)_{q^2>15}/\zeta(\Lambda_b^0 \to \Lambda_c^+\mu^-\overline{\nu}_\mu)_{q^2>7}$	$1.471 \pm 0.096 \pm 0.290$	[97]	⋆	⋆
$\zeta(\bar{B}_s^0 \to K^+\mu^-\overline{\nu}_\mu)_{q^2>7}$	$(3.32 \pm 0 \pm 0.46)$ ps^{-1}	[16, 59]	-	⋆
$\zeta(\bar{B}_s^0 \to D_s^+\mu^-\overline{\nu}_\mu)$	$(9.15 \pm 0 \pm 0.37)$ ps^{-1}	[16, 58]	-	⋆

Table 2: *Inputs to the standard CKM fit. If not stated otherwise: for two errors given, the first is statistical and accountable systematic and the second stands for systematic theoretical uncertainties. The last two columns indicate* R*fit treatment of the input parameters: measurements or parameters that have statistical errors (we include here experimental systematics) are marked in the "GS" column by a star; measurements or parameters that have systematic theoretical errors are marked in the "TH" column by a star. Upper part: parameters used in SM predictions that are obtained from experiment. Lower part: parameters of the SM predictions obtained from theory.*

4 Decay constants

4.1 Light mesons

f_K

Reference	Article	N_f	Mean	Stat	Syst
ETM09	[66]	2	158.1	0.8	3.1
HPQCD07	[67]	2+1	157	0.6	3.3
MILC10	[68]	2+1	156.1	0.4	$^{+0.6}_{-0.9}$
HPQCD13	[69]	2+1+1	155.4	0.2	0.6
ETM14	[70]	2+1+1	155.0	1.4	2.0
Our average			155.57	0.17	0.57

f_K/f_π

Reference	Article	N_f	Mean	Stat	Syst
ETM09	[66]	2	1.210	0.006	0.024
HPQCD/UKQCD07	[67]	2+1	1.189	0.002	0.014
BMW10	[71]	2+1	1.192	0.007	0.013
MILC10	[68]	2+1	1.197	0.002	$^{+0.003}_{-0.007}$
BMW16	[72]	2+1	1.182	0.010	0.026
QCDSF/UKQCD16	[73]	2+1	1.192	0.010	0.026
HPQCD13	[69]	2+1+1	1.1948	0.0015	0.0039
ETM14	[70]	2+1+1	1.188	0.011	0.020
FNAL/MILC17	[74]	2+1+1	1.198	0.0012	$^{+0.0007}_{-0.0021}$
CalLat20	[112]	2+1+1	1.1964	0.0032	0.0060
Our average			1.1973	0.0007	0.0014

The results have been corrected to yield the decay constants defined in QCD (electromagnetic corrections are applied at the level of the branching ratios).

4.2 Charmed mesons

f_{D_s}

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[75]	2	250	5	5
BB19	[120]	2	244	4	2
HPQCD10	[76]	2+1	248.0	1.4	4.5
FNAL-MILC11	[77]	2+1	260.1	8.9	16.2
ChiQCD14	[78]	2+1	254.0	2.2	10.2
RBC-UKQCD17	[79]	2+1	246.4	1.9	$^{+2.5}_{-3.7}$
ETM14	[70]	2+1+1	247.2	3.9	2.2
FNAL/MILC17	[74]	2+1+1	249.9	0.28	0.65
Our average			249.23	0.27	0.65

f_{D_s}/f_D

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[75]	2	1.201	0.007	0.020
FNAL-MILC11	[77]	2+1	1.188	0.014	0.054
HPQCD12	[80]	2+1	1.187	0.004	0.023
RBC-UKQCD17	[79]	2+1	1.1667	0.0077	$^{+0.0090}_{-0.0069}$
ETM14	[70]	2+1+1	1.192	0.019	0.017
FNAL/MILC17	[74]	2+1+1	1.1749	0.0006	0.0033
Our average			1.1782	0.0006	0.0033

4.3 Beauty mesons

f_{B_s}

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[75]	2	228	5	9
ALPHA14	[81]	2	224	14	2
BB19	[120]	2	215	10	$^{+4}_{-7}$
HPQCD11	[82]	2+1	225.0	2.9	5.4
FNAL-MILC11	[77]	2+1	242.0	5.1	21.2
RBC-UKQCD14	[83]	2+1	235.4	5.2	28.1
HPQCD13	[84]	2+1+1	224.0	2.5	7.2
ETM16	[85]	2+1+1	229	3.9	4.8
HPQCD17	[86]	2+1+1	236	2.6	10.0
FNAL/MILC17	[74]	2+1+1	230.7	0.8	1.9
Our average			228.75	0.69	1.87

Ref. [87] is not included due to the significant correlation with HPQCD13, as discussed in Ref. [88]. The same argument holds for Ref. [89] compared to RBC-UKQCD14 (with the additional issue of the uncertainty from the infinite quark limit). Only (subdominant) statistical uncertainties are correlated for HPQCD13 and HPQCD18, and we keep both determinations.

f_{B_s}/f_B

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[75]	2	1.206	0.010	0.026
ALPHA14	[81]	2	1.203	0.062	0.019
FNAL-MILC11	[77]	2+1	1.229	0.013	0.046
RBC-UKQCD14	[83]	2+1	1.223	0.013	0.106
HPQCD13	[84]	2+1+1	1.205	0.004	0.007
ETM16	[85]	2+1+1	1.184	0.018	0.028
HPQCD17	[86]	2+1+1	1.207	0.003	0.006
FNAL/MILC17	[74]	2+1+1	1.2180	0.0033	0.0058
Our average			1.2118	0.0020	0.0058

Ref. [87] is not included due to the significant correlation with HPQCD13, as discussed in Ref. [88]. The same argument holds for Ref. [89] compared to RBC-UKQCD14 (with the additional issue of the uncertainty from the infinite quark limit). Only (subdominant) statistical uncertainties are correlated for HPQCD13 and HPQCD18, and we keep both determinations.

5 Semileptonic form factors

5.1 $K \to \pi \ell \nu$

$f_+^{K\to\pi}(0)$

Reference	Article	N_f	Mean	Stat	Syst
ETM09	[90]	2	0.9560	0.0057	0.0118
MILC12	[91]	2+1	0.9667	0.0023	0.0055
RBC-UKQCD15	[92]	2+1	0.9685	0.0034	0.0019
MILC18	[116]	2+1+1	0.9696	0.0015	0.0023
ETM16	[93]	2+1+1	0.9697	0.0045	0.0036
Our average			0.9675	0.0011	0.0023

5.2 $D \to \pi \ell \nu$

$f_+^{D\to\pi}(0)$

Reference	Article	N_f	Mean	Stat	Syst
HPQCD11	[94]	2+1	0.666	0.021	0.048
ETM17	[95]	2+1+1	0.612	0.035	0.012
FNAL/MILC23	[123]	2+1+1	0.6300	0.0046	0.0060
Our average			0.624	0.004	0.006

5.3 $D \to K \ell \nu$

$f_+^{D\to K}(0)$

Reference	Article	N_f	Mean	Stat	Syst
HPQCD10	[96]	2+1	0.747	0.011	0.034
ETM17	[95]	2+1+1	0.765	0.029	0.012
HPQCD21	[121]	2+1+1	0.7380	0.0037	0.0042
FNAL/MILC23	[123]	2+1+1	0.7452	0.0030	0.0039
Our average			0.742	0.002	0.004

5.4 $\Lambda_b \to p\mu^-\bar{\nu}_\mu$ and $\Lambda_b \to \Lambda_c\bar{\nu}_\mu$

$\zeta(\Lambda_b \to p\mu^-\bar{\nu}_\mu)_{q^2>15}/\zeta(\Lambda_b \to \Lambda_c\mu^-\bar{\nu}_\mu)_{q^2>7}$

Reference	Article	N_f	Mean	Stat	Syst
DLM15	[97]	2+1	1.471	0.096	0.290
Our average			1.471	0.096	0.290

5.5 $B_s^0 \to K^-\mu^+\nu_\mu$ and $B_s^0 \to D_s^-\mu^+\nu_\mu$

$\zeta(B_s^0 \to K^-\mu^+\nu_\mu)_{q^2>7}$

Reference	Article	N_f	Mean	Stat	Syst
FNAL/MILC19	[59]	2+1	3.32	-	0.46
Our average			3.32	-	0.46

$\zeta(B_s^0 \to D_s^-\mu^+\nu_\mu)$

Reference	Article	N_f	Mean	Stat	Syst
HPQCD20	[58]	2+1	9.15	-	0.37
Our average			9.15	-	0.37

6 Meson mixing

6.1 Kaon mixing

$B_K^{\overline{\text{MS}}}(2\text{ GeV})$

Reference	Article	N_f	Mean	Stat	Syst
ETM12	[98]	2	0.531	0.016	0.009
LVdW11	[99]	2+1	0.5572	0.0028	0.0257
BMW11	[100]	2+1	0.5644	0.0059	0.0100
RBC-UKQCD14	[101]	2+1	0.5477	0.002	0.011
SWME15	[102]	2+1	0.537	0.004	0.041
ETM15	[103]	2+1+1	0.5237	0.0131	0.0351
Our average for $B_K^{\overline{\text{MS}}}(2\text{ GeV})$			0.5527	0.0014	0.0090
Our average for $\hat{B}_K$			0.7567	0.0020	0.0123

6.2 $B_{d,s}$ mixing

$\hat{B}_{B_s}$

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[75]	2	1.32	0.04	0.03
HPQCD09	[104]	2+1	1.326	0.018	0.040
RBC-UKQCD14	[89]	2+1	1.22	0.06	0.32
FNAL-MILC16	[105]	2+1	1.443	0.078	0.138
HPQCD19	[119]	2+1+1	1.232	0.017	0.089
Our average			1.313	0.012	0.030

Ref. [104] provide only f_{B_s} and $f_{B_s}\sqrt{\hat{B}_{B_s}}$, and we assumed that the systematics were completely correlated to extract $\hat{B}_{B_s}$.

$\hat{B}_{B_s}/\hat{B}_{B_d}$

Reference	Article	N_f	Mean	Stat	Syst
ETM13	[75]	2	1.007	0.015	0.014
RBC-UKQCD14	[89]	2+1	1.028	0.060	0.100
FNAL-MILC16	[105]	2+1	1.033	0.029	0.048
HPQCD19	[119]	2+1+1	1.008	0.015	0.021
Our average			1.007	0.010	0.014

7 Quark masses

$\bar{m}_c(\bar{m}_c)$

Reference	Article	N_f	Mean	Stat	Syst
HPQCD10	[106]	2+1	1.273	0.0022	0.0120
ChiQCD14	[78]	2+1	1.304	0.0050	0.0449
ALPHA21	[113]	2+1	1.296	0.0088	0.0266
ETM14	[107]	2+1+1	1.3478	0.0027	0.0195
HPQCD14	[108]	2+1+1	1.2715	0.0030	0.0225
HPQCD20	[115]	2+1+1	1.2719	0.0043	0.018
FNAL/MILC/TUMQCD18	[110]	2+1+1	1.273	0.004	0.012
ETM21	[122]	2+1+1	1.339	0.022	$^{+0.030}_{-0.021}$
Our average			1.2977	0.0013	0.012

8 Results of the CKM global fit

8.1 Global fit

Observable	central ± CL ≡ 1σ	± CL ≡ 2σ	± CL ≡ 3σ
A	$0.8215^{+0.0047}_{-0.0082}$	$0.821^{+0.010}_{-0.020}$	$0.821^{+0.015}_{-0.024}$
λ	$0.22498^{+0.00023}_{-0.00021}$	$0.22498^{+0.00052}_{-0.00043}$	$0.22498^{+0.00081}_{-0.00061}$
$\bar{\rho}$	$0.1562^{+0.0112}_{-0.0040}$	$0.1562^{+0.0230}_{-0.0092}$	$0.156^{+0.030}_{-0.014}$
$\bar{\eta}$	$0.3551^{+0.0051}_{-0.0057}$	$0.355^{+0.011}_{-0.015}$	$0.355^{+0.018}_{-0.020}$
J $[10^{-5}]$	$3.115^{+0.047}_{-0.059}$	$3.12^{+0.11}_{-0.15}$	$3.12^{+0.18}_{-0.19}$
$\sin 2\alpha$	$-0.035^{+0.024}_{-0.051}$	$-0.035^{+0.056}_{-0.127}$	$-0.035^{+0.085}_{-0.167}$
$\sin 2\alpha$ (meas. not in the fit)	$-0.046^{+0.031}_{-0.085}$	$-0.046^{+0.062}_{-0.135}$	$-0.046^{+0.091}_{-0.166}$
$\sin 2\beta$	$0.7155^{+0.0079}_{-0.0071}$	$0.715^{+0.018}_{-0.015}$	$0.715^{+0.028}_{-0.023}$
$\sin 2\beta$ (meas. not in the fit)	$0.742^{+0.022}_{-0.024}$	$0.742^{+0.036}_{-0.039}$	$0.742^{+0.047}_{-0.050}$
α [°]	$91.06^{+1.43}_{-0.75}$	$91.1^{+3.6}_{-1.6}$	$91.1^{+4.8}_{-2.5}$
α [°] (meas. not in the fit)	$91.28^{+2.73}_{-0.88}$	$91.3^{+4.0}_{-1.8}$	$91.3^{+4.9}_{-2.6}$
α [°] (dir. meas.)	$86.2^{+3.9}_{-3.5} \| \| -1.0^{+3.3}_{-4.9}$	$86.2^{+12.3}_{-6.6} \| \| -1.0^{+9.4}_{-10.6}$	$86.2^{+21.4}_{-9.4} \| \| -1^{+13}_{-18}$
β [°]	$22.84^{+0.33}_{-0.30}$	$22.84^{+0.73}_{-0.61}$	$22.84^{+1.15}_{-0.94}$
β [°] (meas. not in the fit)	$24.21^{+0.74}_{-1.12}$	$24.2^{+1.4}_{-1.8}$	$24.2^{+1.9}_{-2.3}$
β [°] (dir. meas.)	$22.52^{+0.45}_{-0.44}$	$22.52^{+0.90}_{-0.88}$	$22.5^{+1.4}_{-1.3}$
γ [°]	$66.23^{+0.60}_{-1.43}$	$66.2^{+1.4}_{-3.7}$	$66.2^{+2.1}_{-4.7}$
γ [°] (meas. not in the fit)	$66.29^{+0.72}_{-1.86}$	$66.3^{+1.4}_{-4.2}$	$66.3^{+2.2}_{-5.0}$
γ [°] (dir. meas.)	$65.9^{+3.3}_{-3.5}$	$65.9^{+6.6}_{-7.0}$	$65.9^{+9.9}_{-10.5}$
R_u	$0.3880^{+0.0052}_{-0.0042}$	$0.3880^{+0.0125}_{-0.0095}$	$0.388^{+0.019}_{-0.015}$
R_t	$0.9150^{+0.0045}_{-0.0105}$	$0.9150^{+0.0095}_{-0.0242}$	$0.915^{+0.014}_{-0.031}$
$\bar{\rho}_s$	$-0.00835^{+0.00027}_{-0.00044}$	$-0.00835^{+0.00054}_{-0.00134}$	$-0.00835^{+0.00080}_{-0.00167}$
$\bar{\eta}_s$	$-0.01896^{+0.00035}_{-0.00026}$	$-0.01896^{+0.00083}_{-0.00060}$	$-0.01896^{+0.00115}_{-0.00096}$
$\beta_s \equiv -\arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right)$ [rad]	$0.01882^{+0.00026}_{-0.00028}$	$0.01882^{+0.00059}_{-0.00088}$	$0.01882^{+0.00093}_{-0.00119}$
$\sin 2\beta_s$	$0.03757^{+0.00057}_{-0.00054}$	$0.0376^{+0.0012}_{-0.0017}$	$0.0376^{+0.0019}_{-0.0023}$

(!) means that the quantity was not included in the fit, || indicates the union of the confidence intervals considered.

Observable	central ± CL ≡ 1σ	± CL ≡ 2σ	± CL ≡ 3σ
$\|V_{ud}\|$	$0.974358^{+0.000049}_{-0.000054}$	$0.974358^{+0.000097}_{-0.000121}$	$0.97436^{+0.00014}_{-0.00019}$
$\|V_{us}\|$	$0.22498^{+0.00023}_{-0.00022}$	$0.22498^{+0.00052}_{-0.00043}$	$0.22498^{+0.00081}_{-0.00062}$
$\|V_{ub}\|$	$0.003730^{+0.000044}_{-0.000048}$	$0.00373^{+0.00011}_{-0.00010}$	$0.00373^{+0.00019}_{-0.00015}$
$\|V_{cd}\|$	$0.22484^{+0.00023}_{-0.00021}$	$0.22484^{+0.00052}_{-0.00043}$	$0.22484^{+0.00081}_{-0.00061}$
$\|V_{cs}\|$	$0.973509^{+0.000054}_{-0.000059}$	$0.97351^{+0.00011}_{-0.00013}$	$0.97351^{+0.00016}_{-0.00019}$
$\|V_{cb}\|$	$0.04160^{+0.00020}_{-0.00058}$	$0.04160^{+0.00045}_{-0.00100}$	$0.04160^{+0.00066}_{-0.00121}$
$\|V_{td}\|$	$0.008573^{+0.000046}_{-0.000158}$	$0.00857^{+0.00012}_{-0.00031}$	$0.00857^{+0.00018}_{-0.00037}$
$\|V_{ts}\|$	$0.04088^{+0.00020}_{-0.00066}$	$0.04088^{+0.00043}_{-0.00097}$	$0.04088^{+0.00066}_{-0.00119}$
$\|V_{tb}\|$	$0.9991248^{+0.0000268}_{-0.0000074}$	$0.999125^{+0.000044}_{-0.000017}$	$0.999125^{+0.000052}_{-0.000027}$
$\|V_{ud}\|$ (meas. not in the fit)	$0.974402^{+0.000058}_{-0.000058}$	$0.97440^{+0.00010}_{-0.00013}$	$0.97440^{+0.00015}_{-0.00021}$
$\|V_{us}\|$ (meas. not in the fit)	$0.22535^{+0.00141}_{-0.00039}$	$0.22535^{+0.00418}_{-0.00078}$	$0.2253^{+0.0053}_{-0.0012}$
$\|V_{ub}\|$ (meas. not in the fit)	$0.003673^{+0.000072}_{-0.000084}$	$0.00367^{+0.00015}_{-0.00014}$	$0.00367^{+0.00023}_{-0.00019}$
$\|V_{cb}\|$ (meas. not in the fit)	$0.04178^{+0.00061}_{-0.00078}$	$0.04178^{+0.00090}_{-0.00129}$	$0.0418^{+0.0012}_{-0.0016}$
Δm_d [ps^{-1}] (meas. not in the fit)	$0.534^{+0.033}_{-0.027}$	$0.534^{+0.055}_{-0.060}$	$0.534^{+0.078}_{-0.082}$
Δm_s [ps^{-1}] (meas. not in the fit)	$17.26^{+0.63}_{-0.41}$	$17.26^{+1.87}_{-0.97}$	$17.3^{+2.3}_{-1.3}$
$\|\epsilon_K\|$ [10^{-3}] (meas. not in the fit)	$2.07^{+0.33}_{-0.13}$	$2.07^{+0.41}_{-0.31}$	$2.07^{+0.49}_{-0.37}$
m_t [GeV/c^2] (meas. not in the fit)	$165.0^{+4.1}_{-6.0}$	$165.0^{+8.7}_{-8.2}$	$165.0^{+11.1}_{-9.9}$
B_K (lattice value not in the fit)	$0.772^{+0.076}_{-0.057}$	$0.772^{+0.154}_{-0.081}$	$0.77^{+0.20}_{-0.10}$
f_{B_s}/f_{B_d} (lattice value not in the fit)	$1.231^{+0.037}_{-0.028}$	$1.231^{+0.064}_{-0.066}$	$1.231^{+0.090}_{-0.087}$
f_{B_s} (lattice value not in the fit)	$0.2294^{+0.0035}_{-0.0065}$	$0.2294^{+0.0074}_{-0.0085}$	$0.2294^{+0.0091}_{-0.0101}$
B_{B_s}/B_{B_d} (lattice value not in the fit)	$1.061^{+0.044}_{-0.062}$	$1.061^{+0.089}_{-0.118}$	$1.06^{+0.13}_{-0.15}$
B_{B_s} (lattice value not in the fit)	$1.303^{+0.051}_{-0.055}$	$1.303^{+0.087}_{-0.070}$	$1.303^{+0.104}_{-0.085}$

(!) means that the quantity was not included in the fit, || indicates the union of the confidence intervals considered.

Observable	**central** $\pm$ CL $\equiv 1\sigma$	$\pm$ CL $\equiv 2\sigma$	$\pm$ CL $\equiv 3\sigma$
$\mathcal{B}(B^+ \to \tau\nu)$ $[10^{-4}]$	$0.869\,^{+0.031}_{-0.030}$	$0.869\,^{+0.073}_{-0.062}$	$0.869\,^{+0.110}_{-0.086}$
$\mathcal{B}(B^+ \to \tau\nu)$ $[10^{-4}]$ (meas. not in the fit)	$0.857\,^{+0.035}_{-0.032}$	$0.857\,^{+0.071}_{-0.056}$	$0.857\,^{+0.110}_{-0.079}$
$\mathcal{B}(B^+ \to \mu\nu)$ $[10^{-6}]$	$0.391\,^{+0.013}_{-0.016}$	$0.391\,^{+0.032}_{-0.028}$	$0.391\,^{+0.050}_{-0.039}$
$\mathcal{B}(B^+ \to e\nu)$ $[10^{-11}]$	$0.916\,^{+0.032}_{-0.038}$	$0.916\,^{+0.076}_{-0.068}$	$0.916\,^{+0.116}_{-0.093}$
$\mathcal{B}(B_d \to e^+e^-)$ $[10^{-15}]$	$2.241\,^{+0.096}_{-0.088}$	$2.24\,^{+0.14}_{-0.18}$	$2.24\,^{+0.18}_{-0.21}$
$\mathcal{B}(B_d \to \mu^+\mu^-)$ $[10^{-11}]$	$9.63\,^{+0.38}_{-0.40}$	$9.63\,^{+0.58}_{-0.81}$	$9.63\,^{+0.73}_{-0.95}$
$\mathcal{B}(B_s \to e^+e^-)$ $[10^{-14}]$	$7.68\,^{+0.17}_{-0.31}$	$7.68\,^{+0.39}_{-0.47}$	$7.68\,^{+0.47}_{-0.54}$
$\mathcal{B}(B_s \to \mu^+\mu^-)$ $[10^{-9}]$	$3.25\,^{+0.13}_{-0.10}$	$3.25\,^{+0.19}_{-0.17}$	$3.25\,^{+0.23}_{-0.20}$
$\mathcal{B}(B_s \to \mu^+\mu^-)$ $[10^{-9}]$ (meas. not in the fit)	$3.30\,^{+0.12}_{-0.16}$	$3.30\,^{+0.16}_{-0.22}$	$3.30\,^{+0.19}_{-0.25}$
$\mathcal{B}(D_s \to \tau^+\nu)$	$0.05223\,^{+0.00011}_{-0.00046}$	$0.05223\,^{+0.00023}_{-0.00072}$	$0.05223\,^{+0.00034}_{-0.00084}$
$\mathcal{B}(D_s \to \tau^+\nu)$ (meas. not in the fit)	$0.05205\,^{+0.00028}_{-0.00047}$	$0.05205\,^{+0.00039}_{-0.00059}$	$0.05205\,^{+0.00051}_{-0.00070}$
$\mathcal{B}(D_s \to \mu^+\nu)$ $[10^{-2}]$	$0.5360\,^{+0.0011}_{-0.0047}$	$0.5360\,^{+0.0023}_{-0.0074}$	$0.5360\,^{+0.0034}_{-0.0087}$
$\mathcal{B}(D_s \to \mu^+\nu)$ $[10^{-2}]$ (meas. not in the fit)	$0.5360\,^{+0.0010}_{-0.0059}$	$0.5360\,^{+0.0022}_{-0.0076}$	$0.5360\,^{+0.0034}_{-0.0089}$
$\mathcal{B}(D \to \tau^+\nu)$ $[10^{-3}]$	$1.071\,^{+0.015}_{-0.017}$	$1.071\,^{+0.035}_{-0.036}$	$1.071\,^{+0.050}_{-0.050}$
$\mathcal{B}(D \to \tau^+\nu)$ $[10^{-3}]$ (meas. not in the fit)	$1.064\,^{+0.024}_{-0.014}$	$1.064\,^{+0.035}_{-0.030}$	$1.064\,^{+0.046}_{-0.043}$
$\mathcal{B}(D \to \mu^+\nu)$ $[10^{-3}]$	$0.4009\,^{+0.0037}_{-0.0038}$	$0.4009\,^{+0.0070}_{-0.0060}$	$0.4009\,^{+0.0086}_{-0.0073}$
$\mathcal{B}(D \to \mu^+\nu)$ $[10^{-3}]$ (meas. not in the fit)	$0.4016\,^{+0.0053}_{-0.0028}$	$0.4016\,^{+0.0068}_{-0.0063}$	$0.4016\,^{+0.0082}_{-0.0077}$
$\mathcal{B}(K \to \mu^+\nu)$	$0.63578\,^{+0.00088}_{-0.00097}$	$0.6358\,^{+0.0018}_{-0.0020}$	$0.6358\,^{+0.0027}_{-0.0030}$
$\mathcal{B}(K \to \mu^+\nu)$ (meas. not in the fit)	$0.6361\,^{+0.0015}_{-0.0020}$	$0.6361\,^{+0.0032}_{-0.0044}$	$0.6361\,^{+0.0049}_{-0.0068}$
$\mathcal{B}(K \to e^+\nu)$ $[10^{-4}]$	$0.15706\,^{+0.00022}_{-0.00024}$	$0.15706\,^{+0.00045}_{-0.00049}$	$0.15706\,^{+0.00068}_{-0.00073}$
$\mathcal{B}(K \to e^+\nu)$ $[10^{-4}]$ (meas. not in the fit)	$0.15695\,^{+0.00021}_{-0.00031}$	$0.15695\,^{+0.00046}_{-0.00058}$	$0.15695\,^{+0.00070}_{-0.00084}$
$\mathcal{B}(\tau^+ \to K\nu)$ $[10^{-2}]$	$0.7169\,^{+0.0010}_{-0.0011}$	$0.7169\,^{+0.0021}_{-0.0022}$	$0.7169\,^{+0.0031}_{-0.0033}$
$\mathcal{B}(\tau^+ \to K\nu)$ $[10^{-2}]$ (meas. not in the fit)	$0.7173\,^{+0.0010}_{-0.0011}$	$0.7173\,^{+0.0020}_{-0.0022}$	$0.7173\,^{+0.0031}_{-0.0034}$

(!) means that the quantity was not included in the fit, || indicates the union of the confidence intervals considered.

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[6] We use the same inputs as in Ref. [3], that includes the new measurements of partial branching fractions of inclusive $B \to X_u l\nu$ [4], leading to the inclusive $|V_{ub}| = (4.13 \pm 0.12 \pm 0.14) \times 10^{-3}$. For the exclusive extraction, we consider Ref. [64], together with the new measurement by Belle II of $|V_{ub}|$ from tagged $B \to \pi\ell\nu$, resulting in the exclusive $|V_{ub}| = (3.60 \pm 0.10 \pm 0.12) \times 10^{-3}$. We separate the statistical and systematic uncertainties in both cases. Our average of the two values is obtained using the Educated Rfit approach.

[7] The inclusive value $|V_{cb}| = (42.15 \pm 0.32 \pm 0.39) \times 10^{-3}$ comes from Ref. [10]. The exclusive extraction $|V_{cb}| = (40.08 \pm 0.36 \pm 0.37) \times 10^{-3}$ combines values extracted from $B \to D\ell\nu_\ell$ (BCL) [3,64] and $B \to D^*\ell\nu_\ell$ (BGL only). The latter includes new results from Belle (tagged) [8] and Belle II (untagged) [9]. Lattice QCD inputs for the $B \to D^*\ell\nu_\ell$ form factors come from JLQCD [11], Fermilab MILC [12], and HPQCD [13]; see Ref. [2] for further discussion. We separate the statistical and systematic uncertainties in all cases. Our average of the two values is obtained using the Educated Rfit approach.

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